

REFORMULATION OF MULTI-ATTRIBUTE UTILITY THEORY NORMALIZATION TO HANDLE ASYMMETRIC DATA IN MADM

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Abstract— Multi-Attribute Utility Theory (MAUT) is a widely used multi-attribute decision-making (MADM) method due to its ability to integrate multiple criteria into a single utility value. However, conventional MAUT faces limitations when handling asymmetric data, where standard normalization processes often lead to value distortion and less representative rankings. This study aims to reformulate the normalization function in MAUT to improve adaptability to non-symmetric data distributions and to enhance ranking validity in decision-making. A modification approach called MAUT-A was developed by applying an adaptive normalization mechanism capable of accommodating extreme distributions and outliers by adding Z-score normalization. The performance of MAUT-A was evaluated by comparing the correlation of its ranking results with reference rankings, and the outcomes were benchmarked against conventional MAUT. The experimental findings indicate that conventional MAUT achieved a correlation value of 0.9688 with the reference ranking, while the proposed MAUT-A method achieved a higher correlation of 0.9792. This improvement represents that MAUT-A has better suitability, stability, and reliability in managing asymmetric data. The study contributes by offering a reformulated MAUT framework through adaptive normalization, providing more accurate, stable, and fair ranking outcomes. This approach enhances the validity of MADM applications, particularly in contexts involving asymmetric data distributions.

Keywords: asymmetric data, decision making, madm, maud, normalization reformulation

Intisari— Multi-Attribute Utility Theory (MAUT) merupakan salah satu metode multi-attribute decision-making (MADM) yang banyak digunakan karena kemampuannya mengintegrasikan berbagai kriteria ke dalam satu nilai utilitas. Namun, MAUT konvensional memiliki keterbatasan dalam menangani data yang bersifat asimetris, di mana normalisasi standar sering menimbulkan distorsi nilai dan menghasilkan perankingan yang kurang representatif. Penelitian ini bertujuan merumuskan ulang fungsi normalisasi pada MAUT agar lebih adaptif terhadap distribusi data tidak simetris, sehingga dapat meningkatkan validitas hasil perankingan. Pendekatan modifikasi yang dinamakan MAUT-A dikembangkan dengan menerapkan mekanisme normalisasi adaptif yang mampu mengakomodasi distribusi ekstrim dan outlier dengan menambahkan normalisasi skor Z. Kinerja MAUT-A dievaluasi melalui perbandingan nilai korelasi hasil perankingan dengan peringkat acuan, serta dibandingkan dengan MAUT konvensional. Hasil pengujian menegaskan bahwa MAUT konvensional memperoleh nilai korelasi 0,9688, sedangkan MAUT-A menghasilkan korelasi yang lebih tinggi, yaitu 0,9792. Peningkatan ini merepresentasikan bahwa MAUT-A memiliki kesesuaian, stabilitas, dan reliabilitas yang lebih baik dalam mengelola data asimetris. Penelitian ini memberikan kontribusi berupa reformulasi MAUT melalui normalisasi adaptif yang mampu menghasilkan perankingan lebih akurat, stabil, dan adil. Pendekatan ini memperkuat validitas aplikasi MADM, khususnya pada kasus dengan distribusi data yang tidak seimbang.

Kata Kunci: data asimetris, pengambilan keputusan, madm, maud, reformulasi normalisasi

INTRODUCTION

The multi-attribute decision-making method (MADM) is very important in various domains because it can handle the complexity of decision-making involving multiple conflicting criteria[1], [2]. In the real world, decisions rarely depend on just one factor whether in management, engineering, health, or public policy, various criteria such as cost, quality, risk, and time often need to be considered simultaneously. In MADM, real-world decision-making processes often involve multiple interacting criteria, such as in employee recruitment, performance evaluation, or supplier selection. The main challenge is how to integrate criteria with different levels of importance so that the resulting decision is fair and rational. Relying solely on subjective judgment can lead to biased decisions that are difficult to justify.

MADM allows decision-makers to systematically assess each alternative based on a set of predefined criteria, thereby resulting in more rational, transparent, and accountable decisions[3], [4]. This method not only helps reduce subjectivity in the decision-making process but also provides a structured framework for evaluating alternatives using techniques such as SAW, TOPSIS, AHP, and others. Thus, MADM makes a significant contribution to improving decision quality, especially in situations where quantitative and qualitative data need to be considered simultaneously. One of the main advantages of MADM is its ability to accommodate various criteria simultaneously, both quantitative and qualitative. MADM also provides a systematic and transparent approach, making the decision-making process more objective and accountable.

Multi-Attribute Utility Theory (MAUT) plays an important role as one of the classic methods in multi-criteria decision-making that has been widely used in various fields[5-7]. MAUT allows decision-makers to evaluate and compare various alternatives based on the level of utility or satisfaction provided by each criterion. With a utility-based approach, MAUT is able to quantify the subjective preferences of decision-makers and systematically combine them into a single aggregate value, thus facilitating the selection of the best alternative. As a normative and rational method, MAUT is very suitable for use in situations that require careful consideration of risks, uncertainties, as well as individual or group preferences. MAUT is also flexible in handling criteria with different measurement scales and is able to logically and structurally explain the reasons behind the selection of alternatives. Its main advantage lies in

its ability to combine the subjective preferences of decision-makers with objective data through a structured utility function. MAUT can also handle various types of criteria with different measurement scales, as well as consider uncertainty and risk in the evaluation process[8]. Furthermore, this method provides transparent results that can be logically explained, making it easier to justify the decisions made. The flexibility and thoroughness of the analysis offered by MAUT make it widely used in various fields.

In the conventional normalization process in the MAUT method, a major challenge arises when faced with asymmetric data, which is data that is unevenly distributed or skewed to one side. MAUT normalization generally assumes that data is linear and evenly distributed, so when confronted with asymmetric data, the scoring results can become biased or not reflect the actual preferences[9]. This has the potential to produce less accurate utility values and decrease the validity of decision-making, especially if there are criteria with outliers or a very wide data distribution. This inconsistency can cause the weight of an alternative to be too high or too low, depending on how extreme data affects the normalization scale. As a result, alternatives that are actually less viable may appear more favorable, and vice versa. Therefore, in the face of asymmetric data, a more adaptive normalization approach is needed, such as the use of non-linear transformations or data distribution-based normalization methods, so that the final results more accurately reflect realistic conditions and preferences.

The implementation of a normalization scheme that is unable to adequately handle asymmetric data can lead to significant distortion in ranking results using the MAUT method. This distortion occurs because extreme values or outliers in the data can dominate the scaling process, exaggerating the differences between alternatives that are actually not that significant. As a result, alternatives with extreme values may appear to be much better or, conversely, significantly worse compared to others, whereas in reality the differences are not that critical. This distortion damages the integrity of the decision-making process because the ranking results become unrepresentative of the real preferences and conditions. The best alternative may not be selected because it is overshadowed by another alternative that only appears to be better due to improper normalization schemes. Therefore, it is important to use a more robust normalization approach to asymmetric data, so that the evaluation process of alternatives in MAUT is more accurate, fair, and reliable.

The aim of this research is to propose a new approach in the normalization process of MADM, particularly within the MAUT framework, that can accommodate the characteristics of asymmetric data. The novelty of this research lies in redefining the conventional normalization scheme that has been sensitive to outliers and imbalanced data distribution, which often results in unstable or biased alternative rankings. By developing a normalization method that is adaptive to the shape of data distribution, it is expected that the final results of the decision-making process will be more stable, accurate, and reliable. This approach is anticipated to enhance the quality and reliability of MADM methods in various real-world applications that often involve data with asymmetric distributions.

The advantages of this approach are tested through its application in real case studies involving attribute data with a high degree of asymmetry, and the results show an improvement in ranking accuracy as well as decision result consistency. This research contributes to the development of a more robust and applicable MAUT method in decision support systems, particularly for cases involving data with non-normal distribution.

MATERIALS AND METHODS

MAUT with Asymmetric Distribution-based Normalization

MAUT with asymmetric distribution-based normalization using z-score normalization is a multi-criteria decision-making method that adapts the normalization process within the MAUT framework to address data with asymmetric distributions. Z-score normalization is used to transform the data of each criterion based on the mean and standard deviation, thereby converting the data into values that indicate how far each data point deviates from the average in terms of standard deviations[10]-[12]. This approach is effective in reducing the influence of outliers and the imbalance of data distribution, as extreme values no longer dominate the normalization process like in conventional methods that only use the minimum and maximum range. Thus, z-score normalization in MAUT helps generate more stable, accurate, and reliable utility values, supporting more objective decision-making that is representative of the actual data conditions, which are often asymmetric.

MAUT-A using Z-Score Normalization is a multi-criteria decision-making method that adapts the normalization process within the MAUT framework to handle data with asymmetric

distributions. Z-score normalization is used to transform the data of each criterion based on the mean and standard deviation, thereby changing the data into values that indicate how far each data point deviates from the mean in standard deviation units. This approach is effective in reducing the influence of outliers and imbalances in data distribution, as extreme values no longer dominate the normalization process as in conventional methods that only use the minimum and maximum range. Thus, the z-score normalization in MAUT helps to produce utility values that are more stable, accurate, and reliable, thereby supporting more objective and representative decision-making in relation to the actual data conditions that are often asymmetric.

MAUT-A is a multi-criteria decision-making approach that modifies the MAUT method to address data challenges with asymmetric distribution. The MAUT-A approach produces evaluations that are more stable, accurate, and reliable in the context of asymmetric data. The stages in the MAUT-A method are as follows.

The decision matrix is an initial data representation in the MAUT-A method, which contains the criterion values for each alternative to be evaluated[13]. The decision matrix is created using the following equation.

$$X = [x_{ij}]_{m \times n} \quad (1)$$

Equation (1) represents an $m \times n$ matrix, where m denotes the number of rows and n denotes the number of columns. The notation x_{ij} indicates the matrix element in the i -th row and j -th column.

Normalization of the decision matrix is the initial normalization of the MAUT-A method, which is the original process of normalizing the MAUT method. This technique uses min-max normalization aimed at converting the original data values into the same scale so that all criteria can be compared equitably even though they have different units or scales[14]. The normalization of the decision matrix is calculated using the following equation.

$$r_{ij}^* = \frac{x_{ij} - \min_i x_{ij}}{\max_i x_{ij} - \min_i x_{ij}}; \text{benefit criteria} \quad (2)$$

$$r_{ij}^* = 1 + \frac{\min_i x_{ij} - x_{ij}}{\max_i x_{ij} - \min_i x_{ij}}; \text{cost criteria} \quad (3)$$

Equations (2) and (3) define the normalization process in multi-criteria decision making by considering the differences between benefit and

cost criteria. The value r_{ij}^* represents the normalized value, $\min_i x_{ij}$ is the minimum value of column i , and $\max_i x_{ij}$ is the maximum value of column i .

The average criterion score is a statistical measure used to describe the general tendency or average performance of a criterion based on the evaluation of all alternatives being assessed[15]. This average score is useful to provide an overview of how well the criterion is generally met by all alternatives, calculated using the following equation.

$$\mu_j = \frac{1}{m} \sum_{i=1}^m x_{ij} \quad (4)$$

The value of μ_j from equation (4) represents the average value of the j -th criterion, while m indicates the total number of alternatives evaluated.

The standard deviation value of the criteria is a statistical measure that indicates the extent of variation or dispersion of values among alternatives on a specific criterion in the decision matrix[16]. Standard deviation is used to see whether the values within one criterion are widely spread out or tend to be close to each other, calculated using the following equation.

$$\sigma_j = \sqrt{\frac{1}{m} \sum_{i=1}^m (x_{ij} - \mu_j)^2} \quad (5)$$

The value of σ_j from Equation (5) represents the standard deviation of the j -th criterion.

Z-score normalization is a data transformation method that changes original values into standard scores based on their statistical distribution[17]. Z-score normalization is used to standardize values from various criteria that may have different units and scales, especially when the data has an asymmetric distribution or contains outliers, calculated using the following equation.

$$r_{ij} = \left| \frac{x_{ij} - \mu_j}{\sigma_j} \right| \quad (6)$$

The value r_{ij} from Equation (6) represents the value normalized using Z-score normalization. The use of the absolute value aims to ensure that the standardization result is always non-negative, so the focus is only on the magnitude of deviation without considering the direction.

The average normalized value is a combination of the values from the 2 normalizations

that have been performed. This average normalized value aims to understand the relative position of the normalized data in relation to the overall distribution and to evaluate how balanced the spread of values among alternatives is within each criterion[18]. The average normalized value is calculated using the following equation.

$$r_{ij}^{average} = \frac{r_{ij}^* + r_{ij}}{2} \quad (7)$$

The value $r_{ij}^{average}$ is the average value of two normalizations in the MAUT-A method.

The utility value is a numerical representation of the level of satisfaction or preference for each alternative based on the normalized criteria[19]. Utility values can quantify and compare alternatives objectively, making it easier to determine the best choice based on established preferences and priorities. The utility value is calculated using the following equation.

$$u_{ij} = \frac{e((r_{ij}^{average})^2) - 1}{1.71} \quad (8)$$

The u_{ij} value from equation (8) is a form of nonlinear transformation used to measure the level of utility or the relative contribution of the i -th alternative to the j -th criterion.

The final utility value is the result of the aggregation of the utility values of each criterion that have been weighted according to their level of importance in the multi-criteria decision-making process[20], [21]. This value reflects the overall score of each alternative, indicating how well the alternative meets all criteria simultaneously. The final utility value is calculated using the following equation.

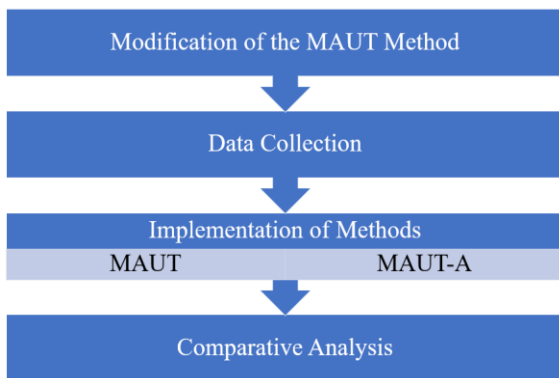
$$A_j = \sum_{i=1}^n u_{ij} * w_j \quad (9)$$

The A_j value from Equation (9) explains the process of calculating the aggregate value or final score for the i -th alternative in a multi-criteria decision-making (MCDM) problem.

MAUT-A is an innovative approach in multi-criteria decision making that addresses the limitations of conventional methods in handling data with asymmetric distributions. This approach makes the process of calculating utility and ranking alternatives more reliable, especially when the data contains uneven values.

Research Stage

The stages of research are a systematic process aimed at discovering, developing, and proving new knowledge through scientific approaches [13], [22], [23]. In an effort to achieve these goals, research is carried out through a series of interconnected and structured stages. Each stage is designed to ensure that the research process is carried out logically, objectively, and can be scientifically accountable. Generally, the stages of research include problem identification, data collection and processing, and result analysis. Organizing these stages is very important for the research to yield valid and relevant findings concerning the issues being examined. The stages of research conducted are displayed in Figure 1.



Source: (Research Result, 2025)
Figure 1. Research Stages

Figure 1 illustrates the research flow, which consists of four main stages. The first stage is the modification of the MAUT method, focusing on the development or refinement of the basic approach to suit the research needs. This is followed by data collection, which serves as the basis for applying the method in the analysis process. In the next stage, method implementation is carried out, involving the application of the conventional MAUT method and the modified MAUT-A version to obtain relevant calculation results. The final stage is a comparative analysis, where the results of both methods are systematically compared to evaluate the advantages, disadvantages, and consistency of each approach in supporting decision-making. Through this flow, the research aims to produce a more comprehensive evaluation of the effectiveness of the methods used.

Employee recruitment criteria are highly relevant to be used as a case study for the application of the MAUT-A approach because the recruitment process is essentially a complex multi-criteria decision-making problem. The selection of candidates does not rely solely on a single factor but

involves various criteria. The values of these criteria are often heterogeneous compared to the criteria used. This aligns with the MAUT-A framework, which is designed to normalize, weight, and aggregate diverse data on a uniform scale.

RESULTS AND DISCUSSION

Modification of the MAUT Method

This research offers a new approach in the normalization process to address the issue of asymmetric data in multi-criteria decision-making, named the MAUT-A method. This reformulation is designed to minimize the distortion of utility values that often occurs due to data distribution imbalance, thus able to produce fairer and more representative alternative evaluations. By considering sensitivity to extreme values, this approach enhances the accuracy of utility calculations and strengthens the validity of the final rankings. The proposed method also maintains result stability when applied to various types of data, making it a more flexible and adaptive solution compared to traditional MAUT approaches.

This approach integrates the principles of dynamic scaling in the normalization process, allowing each criterion to adjust its value range proportionally to the characteristics of data distribution. Thus, the contribution of each attribute to the final utility value becomes more balanced, especially when there are extreme differences between minimum and maximum values. Moreover, this method is capable of maintaining the integrity of preference relationships among alternatives without compromising sensitivity to performance differences of each criterion. Testing in various case scenarios shows that this approach not only improves reliability in decision-making but also provides a strong methodological foundation for further development within the MAUT framework.

Data Collection

Data collection is a crucial stage in research because it serves as the foundation for the analysis process and drawing valid conclusions. At this stage, data is collected systematically according to the needs and objectives of the research by using assessment data in employee recruitment selection. The data for this research criterion is based on six main criteria that are considered relevant in the employee recruitment selection process, namely English test scores, psychological test scores, educational qualifications, work experience, interview results, and technical skills. The data criteria used in this study are presented in Table 1.

Table 1. Criteria Data for Employee Recruitment Selection

Criteria Code	Criteria Name	Description	Weight
C1	English Test Scores	The test scores of the candidates' English language proficiency reflect their global communication skills.	0.2
C2	Psychological Test Scores	The results of the psychological test that assesses the candidate's personality aspects, logic, and emotional stability.	0.2
C3	Educational Qualifications	The candidate's highest level of formal education (High School, Diploma, Bachelor's, Master's).	0.2
C4	Work Experience	Number of years of work experience relevant to the position applied for.	0.15
C5	Interview Results	Assessment from the HR team or user regarding the candidate's performance during the interview.	0.15
C6	Technical Skills	The practical or technical skill score of the candidate aligns with the requirements of the job position.	0.1

Source : (Research Result, 2025)

The assessment process is carried out objectively by the authorized parties and has gone through verification stages to ensure the accuracy and validity of the data before being used in decision analysis. The assessment data in employee recruitment selection is shown in table 2.

Table 2. The Assessment Data in Employee Recruitment Selection

Candidate Name	C1	C2	C3	C4	C5	C6
Employee Candidate A	530	78	3	4	80	8.8
Employee Candidate B	545	82	4	5	85	9.2
Employee Candidate C	490	75	2	3	78	8

Candidate Name	C1	C2	C3	C4	C5	C6
Employee Candidate D	475	70	3	2	65	7.5
Employee Candidate E	540	80	4	6	90	9.5
Employee Candidate F	510	68	3	4	70	8.5
Employee Candidate G	525	85	4	5	88	9
Employee Candidate H	545	85	4	6	90	9.5

Source : (Research Result, 2025)

The data source in this research was obtained through an internal evaluation process conducted by the company's recruitment team on eight candidates who participated in the employee selection process. Data were collected based on six main criteria considered relevant and strategic in assessing the eligibility of prospective employees, namely English test scores, psychological test scores, educational qualifications, work experience, interview results, and technical abilities. Assessments were carried out directly by the interviewers and examiners from each field, using standard instruments that have been tailored to the needs of the applied positions. All data obtained are the result of actual measurements during the selection process and have been verified to ensure validity and reliability before being used in the decision analysis stage. The ranking results of the companies based on internal evaluation of all assessment criteria are displayed in Table 3.

Table 3. Alternative Ranking Results

Candidate Name	Final Value	Rank
Employee Candidate B	46	1
Employee Candidate G	44	2
Employee Candidate A	37	3
Employee Candidate E	36	4
Employee Candidate C	28	5
Employee Candidate F	27	6
Employee Candidate H	24	7
Employee Candidate D	19	8

Source : (Research Result, 2025)

The ranking results obtained from the company's internal evaluation serve as the primary reference in the validation process of the decision-making methods applied in this research. The ranking reflects the subjective assessment made by the company based on experience, professional intuition, and managerial considerations of the candidates' performance across all selection criteria. By comparing the ranking results from the



proposed methods (such as MAUT or MAUT-A) with the actual ranking from the company, the researchers can evaluate the level of consistency, accuracy, and relevance of the approaches used.

Implementation of Method

The implementation of the methods in this research is conducted to process candidate data based on six predetermined selection criteria, with the aim of producing an objective and accountable final ranking. The main method used is MAUT, which is then modified with a normalization reformulation approach to accommodate asymmetric data and improve the accuracy of utility calculations. Each criterion data is normalized according to its characteristics, and then its utility value is calculated based on the established weights. The calculation results from the conventional MAUT method are compared with the MAUT-A method to observe the differences and the level of superiority of the new approach. This stage is the core of the decision-making process, where the final results will be validated against the actual rankings from the company as a benchmark for the validity of the method used.

a) Implementation of the MAUT Method

The implementation of the MAUT method is carried out through a series of systematic stages to convert raw data into information that can be used for decision-making. The first step is data normalization for each criterion, which aims to equalize the assessment scale so that it can be fairly compared, created using (1).

$$X = \begin{bmatrix} 530 & 78 & 3 & 4 & 80 & 8.8 \\ 545 & 82 & 4 & 5 & 85 & 9.2 \\ 490 & 75 & 2 & 3 & 78 & 8 \\ 475 & 70 & 3 & 2 & 65 & 7.5 \\ 540 & 80 & 4 & 6 & 90 & 9.5 \\ 510 & 68 & 3 & 4 & 70 & 8.5 \\ 525 & 85 & 4 & 5 & 88 & 9 \\ 480 & 72 & 2 & 3 & 72 & 8.7 \end{bmatrix}$$

After the decision matrix is created, the next step is to calculate the normalization values to bring the values into the same scale, the normalization using the MAUT method is calculated using (2).

$$r_{11}^* = \frac{x_{11} - \min_i x_{ij}}{\max_i x_{ij} - \min_i x_{ij}} = \frac{530 - 475}{545 - 475} = \frac{55}{70} = 0.7857$$

The results of the total normalization values calculated using (2) are shown in table 4.

Table 4. Normalization Results of the MAUT Method

Candidate Name	C1	C2	C3	C4	C5	C6
Employee Candidate A	0.7857	0.5882	0.5000	0.6667	0.6522	0.7647
Employee Candidate B	1.0000	0.8235	1.0000	1.0000	0.8696	1.0000
Employee Candidate C	0.2143	0.4118	0.0000	0.3333	0.5652	0.2941
Employee Candidate D	0.0000	0.1176	0.5000	0.0000	0.0000	0.0000
Employee Candidate E	0.9286	0.7059	1.0000	0.0000	0.6522	0.2941
Employee Candidate F	0.5000	0.0000	0.5000	0.6667	0.2174	0.5882
Employee Candidate G	0.7143	1.0000	1.0000	1.0000	1.0000	0.8824
Employee Candidate H	0.0714	0.2353	0.0000	0.3333	0.3043	0.1765

Source : (Research Result, 2025)

Calculating the partial utility value of each alternative for each criterion. This utility value indicates how well an alternative performs on a specific criterion after undergoing normalization, and a certain utility function is calculated using (7).

$$u_{11} = \frac{e((r_{11}^*)^2) - 1}{1.71} = \frac{e((0.7857)^2) - 1}{1.71}$$

$$u_{11} = \frac{0.854003}{1.71} = 0.4994$$

The overall results of the utility value calculations for each existing criterion using (7) are shown in Table 5.

Table 5. Utility Score Results of the MAUT Method

Candidate Name	C1	C2	C3	C4	C5	C6
Employee Candidate A	0.4994	0.2418	0.1661	0.3273	0.3100	0.4647
Employee Candidate B	1.0048	0.5674	1.0048	1.0048	0.6608	1.0048
Employee Candidate C	0.0275	0.1081	0.0000	0.0687	0.2201	0.0528
Employee Candidate D	0.0000	0.0082	0.1661	0.0000	0.0000	0.0000
Employee Candidate E	0.8003	0.3777	1.0048	0.0000	0.3100	0.0528
Employee Candidate F	0.1661	0.0000	0.1661	0.3273	0.0283	0.2418
Employee Candidate G	0.3893	1.0048	1.0048	1.0048	1.0048	0.6891
Employee Candidate H	0.0030	0.0333	0.0000	0.0687	0.0568	0.0185

Source : (Research Result, 2025)

Calculating the final utility value in the MAUT method is done to determine the final score of each

alternative, based on the multiplication of the utility value of the alternative with the criterion weight.

The final utility value is calculated using (8).

$$A_1 = \sum_{j=1}^6 u_{1j} * w_j = (u_{11} * w_1) + (u_{12} * w_2) + (u_{13} * w_3) + (u_{14} * w_4) + (u_{15} * w_5) + (u_{16} * w_6)$$

$$A_1 = (0.4994 * 0.2) + (0.2418 * 0.2) + (0.1661 * 0.2) + (0.3273 * 0.15) + (0.3100 * 0.15) + (0.4647 * 0.15)$$

$$A_1 = (0.09988) + (0.04835) + (0.03322) + (0.04909) + (0.04650) + (0.04647) = 0.27705$$

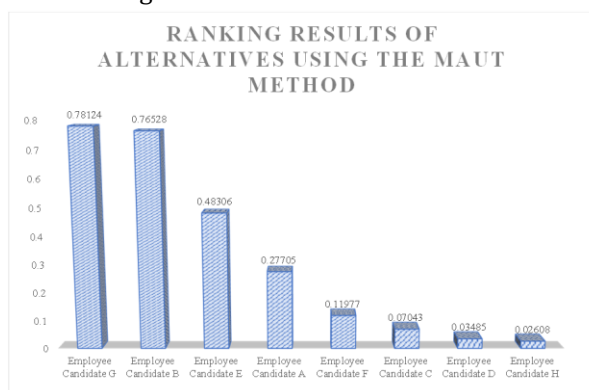
The overall result of the final utility value calculations for each alternative in the MAUT method is shown in table 6.

Table 6. Final Utility of the MAUT Method

Candidate Name	Final Value
Employee Candidate A	0.27705
Employee Candidate B	0.76528
Employee Candidate C	0.07043
Employee Candidate D	0.03485
Employee Candidate E	0.48306
Employee Candidate F	0.11977
Employee Candidate G	0.78124
Employee Candidate H	0.02608

Source : (Research Result, 2025)

The total utility value obtained from the MAUT method reflects the overall performance of each candidate based on six predetermined criteria, which are weighted according to their level of importance. The ranking results of alternatives based on the final values of the MAUT method are shown in Figure 2.



Source : (Research Result, 2025)

Figure 2. Ranking Results of Alternatives using the MAUT Method

The calculation results using the Multi-Attribute Utility Theory (MAUT) method yielded the ranking of the eight employee candidates based on their utility values. The candidate with the highest

utility value is Employee Candidate G with a score of 0.78124, followed by Employee Candidate B with a score of 0.76528. Both demonstrate excellent performance based on the established assessment criteria. In third place is Employee Candidate E with a value of 0.48306, which still falls within the medium value category. Next, Employee Candidate A scored 0.27705, followed by Employee Candidate F with a score of 0.11977. The three other candidates scored lower utility, namely Employee Candidate C (0.07043), Employee Candidate D (0.03485), and Employee Candidate H (0.02608), indicating that they have a relatively low fit to the criteria used in the selection process. These results suggest that candidates G and B are the most optimal choices to consider in the recruitment process, as their scores are well above the other candidates. This assessment process demonstrates the MAUT method's ability to integrate various criteria into a single utility score that can be used for objective and structured decision-making.

b) Implementation of the MAUT-A Method

The implementation of the MAUT-A method is an application of a modified version of the MAUT method designed to handle data that is asymmetric or has an uneven distribution across each criterion.

The decision matrix is the initial data representation in the MAUT-A method, which contains the criterion values for each alternative that will be evaluated. The decision matrix is created using (1).

$$X = \begin{bmatrix} 530 & 78 & 3 & 4 & 80 & 8.8 \\ 545 & 82 & 4 & 5 & 85 & 9.2 \\ 490 & 75 & 2 & 3 & 78 & 8 \\ 475 & 70 & 3 & 2 & 65 & 7.5 \\ 540 & 80 & 4 & 6 & 90 & 9.5 \\ 510 & 68 & 3 & 4 & 70 & 8.5 \\ 525 & 85 & 4 & 5 & 88 & 9 \\ 480 & 72 & 2 & 3 & 72 & 8.7 \end{bmatrix}$$

Normalization of the decision matrix is the initial normalization of the MAUT-A method, which is the preliminary process of normalizing the MAUT method. This technique uses min-max normalization aimed at converting the original data values into the same scale so that all criteria can be compared fairly even though they have different units or scales. The normalization of the decision matrix is calculated using (2).

$$r_{11}^* = \frac{x_{11} - \min_i x_{ij}}{\max_i x_{ij} - \min_i x_{ij}} = \frac{530 - 475}{545 - 475} = \frac{55}{70} = 0.7857$$

The results of the total normalization linier values calculated using (2) are shown in table 7.



Table 7. Normalization Results of the MAUT -A Method

Candidate Name	C1	C2	C3	C4	C5	C6
Employee Candidate A	0.7857	0.5882	0.5000	0.6667	0.6522	0.7647
Employee Candidate B	1.0000	0.8235	1.0000	1.0000	0.8696	1.0000
Employee Candidate C	0.2143	0.4118	0.0000	0.3333	0.5652	0.2941
Employee Candidate D	0.0000	0.1176	0.5000	0.0000	0.0000	0.0000
Employee Candidate E	0.9286	0.7059	1.0000	0.0000	0.6522	0.2941
Employee Candidate F	0.5000	0.0000	0.5000	0.6667	0.2174	0.5882
Employee Candidate G	0.7143	1.0000	1.0000	1.0000	1.0000	0.8824
Employee Candidate H	0.0714	0.2353	0.0000	0.3333	0.3043	0.1765

Source : (Research Result, 2025)

The average criterion score is a statistical measure used to describe the general tendency or average performance of a criterion based on the evaluation of all assessed alternatives. This average score is useful for providing an overview of how well the criterion is generally met by all alternatives, which is calculated using (3).

$$\mu_1 = \frac{1}{8} \sum_{i=1}^8 x_{i1} = \frac{1}{8} (x_{11} + x_{21} + x_{31} + x_{41} + x_{51} + x_{61} + x_{71} + x_{81})$$

$$\mu_1 = \frac{1}{8} (530 + 545 + 490 + 475 + 540 + 510 + 525 + 545) = 513.75$$

The overall results of the average criteria value calculations using the MAUT-A method shown in (3) are displayed in table 8.

Table 8. The Average Score of the Criteria of the MAUT -A Method

	C1	C2	C3	C4	C5	C6
μ_j	513.75	77.125	3.25	3.625	78.25	8.4

Source : (Research Result, 2025)

The standard deviation criterion is a statistical measure that indicates the level of variation or dispersion of values among alternatives based on specific criteria in the decision matrix. Standard deviation is used to see whether the values in one criterion are widely spread or tend to be close

together, calculated using (4).

$$\sigma_1 = \sqrt{\frac{1}{8} \sum_{i=1}^8 (x_{i1} - \mu_1)^2} = \sqrt{\frac{1}{8} (6251.5625)}$$

$$\sigma_1 = \sqrt{781.4453} = 27.3576$$

The overall results of the standard deviation value calculations using the MAUT-A method shown in (4) are displayed in table 9.

Table 9. Standard Deviation in the MAUT -A Method

	C1	C2	C3	C4	C5	C6
σ_j	27.357	6.3332	0.82	1.2183	8.0739	0.62
	6		92			65

Source : (Research Result, 2025)

Z-score normalization is a data transformation method that converts original values into standard scores based on their statistical distribution. Z-score normalization is used to standardize values from various criteria that may have different units and scales, especially when the data has asymmetric distribution or contains outliers, which are calculated using (5).

$$r_{11} = \left| \frac{x_{11} - \mu_1}{\sigma_1} \right| = \left| \frac{530 - 513.75}{27.3576} \right| = \left| \frac{16.25}{27.3576} \right| = 0.59399$$

The results of the total Z-score normalization values calculated using (5) are shown in table 10.

Table 10. Z-Score Normalization Results of the MAUT -A Method

Candidate Name	C1	C2	C3	C4	C5	C6
Employee Candidate A	0.5940	0.1382	0.3015	0.3078	0.2167	0.6385
Employee Candidate B	1.1423	0.7698	0.9045	1.1286	0.8360	1.2769
Employee Candidate C	0.8681	0.3355	1.5076	0.5130	0.0310	0.6385
Employee Candidate D	1.4164	1.1250	0.3015	1.3338	1.6411	1.4366
Employee Candidate E	0.9595	0.4540	0.9045	1.3338	0.2167	0.6385
Employee Candidate F	0.1371	1.4408	0.3015	0.3078	1.0218	0.1596
Employee Candidate G	0.4112	1.2434	0.9045	1.1286	1.2076	0.9577
Employee Candidate H	1.2337	0.8092	1.5076	0.5130	0.7741	0.9577

Source : (Research Result, 2025)

The normalized average value is a combination of the values from the two normalizations that have

been performed. This normalized average value aims to understand the relative position of the normalized data

in relation to the overall distribution and to evaluate how balanced the dispersion of values is among alternatives in each criterion. The normalized average value is calculated using (6).

$$r_{11}^{average} = \frac{r_{11}^* + r_{11}}{2} = \frac{0.7857 + 0.5940}{2} = 0.6898$$

The results of the total normalized average value calculated using (6) are shown in table 11.

Table 11. Z Normalized Average Value Result of the MAUT-A Method

Candidate Name	C1	C2	C3	C4	C5	C6
Employee Candidate A	0.6898	0.3632	0.4008	0.4872	0.4345	0.7016
Employee Candidate B	1.0711	0.7966	0.9523	1.0643	0.8528	1.1385
Employee Candidate C	0.5412	0.3736	0.7538	0.4232	0.2981	0.4663
Employee Candidate D	0.7082	0.6213	0.4008	0.6669	0.8205	0.7183
Employee Candidate E	0.9440	0.5799	0.9523	0.6669	0.4345	0.4663
Employee Candidate F	0.3185	0.7204	0.4008	0.4872	0.6196	0.3739
Employee Candidate G	0.5628	1.1217	0.9523	1.0643	1.1038	0.9200
Employee Candidate H	0.6525	0.5223	0.7538	0.4232	0.5392	0.5671

Source : (Research Result, 2025)

Calculating the partial utility value of each alternative for each criterion. This utility value indicates how well an alternative performs on a specific criterion after undergoing normalization, and a certain utility function is calculated using (7).

$$u_{11} = \frac{e((r_{11}^{average})^2) - 1}{1.71} = \frac{e((0.6898)^2) - 1}{1.71}$$

$$u_{11} = \frac{0.60945}{1.71} = 0.3564$$

The overall results of the utility value calculations for each existing criterion using (7) are shown in table 12.

Table 12. Utility Score Result of the MAUT-A Method

Candidate Name	C1	C2	C3	C4	C5	C6
Employee Candidate A	0.3564	0.0825	0.1019	0.1567	0.1215	0.3719
Employee Candidate B	1.2572	0.5183	0.8634	1.2304	0.6254	1.5527
Employee Candidate C	0.1990	0.0876	0.4474	0.1147	0.0543	0.1420
Employee Candidate D	0.3809	0.2755	0.1019	0.3275	0.5618	0.3948
Employee Candidate E	0.8410	0.2338	0.8634	0.3275	0.1215	0.1420
Employee Candidate F	0.0625	0.3979	0.1019	0.1567	0.2737	0.0878
Employee Candidate G	0.2179	1.4733	0.8634	1.2304	1.3928	0.7786
Employee Candidate H	0.3104	0.1834	0.4474	0.1147	0.1973	0.2218

Source : (Research Result, 2025)

Calculating the final utility value in the MAUT-A method is done to determine the final score of each alternative, based on the multiplication of the utility value of the alternative with the criterion weight. The final utility value is calculated using (8).

$$A_1 = \sum_{j=1}^6 u_{1j} * w_j = (u_{11} * w_1) + (u_{12} * w_2) + (u_{13} * w_3) + (u_{14} * w_4) + (u_{15} * w_5) + (u_{16} * w_6)$$

$$A_1 = (0.3564 * 0.2) + (0.0825 * 0.2) + (0.1019 * 0.2) + (0.1567 * 0.15) + (0.1215 * 0.15) + (0.3719 * 0.15)$$

$$A_1 = (0.07128) + (0.01649) + (0.02038) + (0.02350) + (0.01822) + (0.03719)$$

$$A_1 = 0.18707$$

The overall result of the final utility value

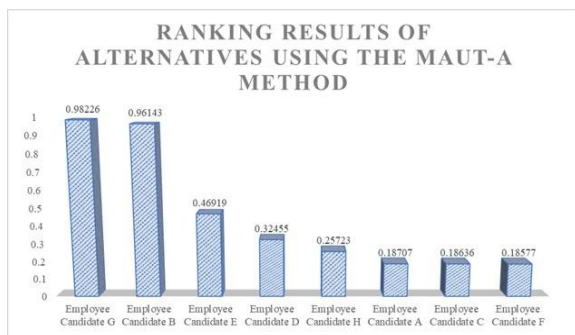
calculations for each alternative in the MAUT-A method is shown in table 13.

Table 13. Final Utility of the MAUT-A Method

Candidate Name	Final Value
Employee Candidate A	0.18707
Employee Candidate B	0.96143
Employee Candidate C	0.18636
Employee Candidate D	0.32455
Employee Candidate E	0.46919
Employee Candidate F	0.18577
Employee Candidate G	0.98226
Employee Candidate H	0.25723

Source : (Research Result, 2025)

The total utility value obtained from the MAUT-A method reflects the overall performance of each candidate based on six predetermined criteria, which are weighted according to their level of importance. The ranking results of alternatives based on the final values of the MAUT-A method are shown in Figure 3.



Source : (Research Result, 2025)

Figure 3. Ranking Results of Alternatives using the MAUT-A Method

The results of the calculations using the MAUT-A method yielded a ranking of eight employee candidates based on their aggregate utility scores. Employee Candidate G occupies the top position with a score of 0.98226, followed by Employee Candidate B with a score of 0.96143. Both candidates demonstrate a very high level of alignment with the established selection criteria, making them the most recommended candidates for acceptance. In the next ranking, Employee Candidate E achieved a score of 0.46919, showing a medium performance that can still be considered. This is followed by Employee Candidate D (0.32455) and Employee Candidate H (0.25723), who have lower utility scores but still show potential in certain aspects. Meanwhile, the three candidates with the lowest scores are Employee Candidate A (0.18707), Employee Candidate C (0.18636), and Employee Candidate F (0.18577), which indicate the lowest level of fit in this selection. Overall, these results show that the MAUT-A method is effective in providing a clear and structured ranking of the available alternatives, allowing decision-makers to choose the most optimal candidates based on a quantitative and multi-criteria approach.

c) Discussion

A comparative analysis was conducted to evaluate the performance differences between the conventional MAUT method and the modified MAUT-A method. The aim of this analysis is to see how much the reformulation at the normalization stage in MAUT-A can provide more accurate, fair, and representative results compared to the standard MAUT, particularly when applied to data that has asymmetric distribution characteristics. By comparing the final results of both methods, a more comprehensive picture of the advantages, disadvantages, and relevance of applying each

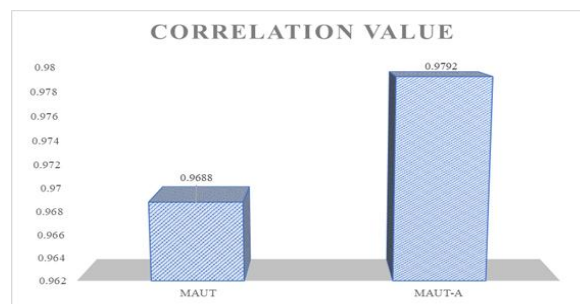
method in the context of multi-criteria decision making is obtained. In the analysis process, the total utility values of each candidate were calculated using both methods and then the ranking results were compared. This comparison highlights not only the differences in preference order but also observes the sensitivity level of each method to variations in values for each criterion. MAUT-A, which integrates a reformulation of normalization, shows better capability in accommodating uneven data distributions, resulting in more proportional assessments. Meanwhile, MAUT tends to produce results that are less responsive to minor differences between alternatives. The results of this comparison provide an important foundation for selecting the most suitable method, especially in contexts requiring attention to detail and fairness in evaluating alternative performance. The results of the alternative ranking comparison are shown in table 14.

Table 14. Alternative Ranking Result

Candidate Name	C1	C2	C3
Employee Candidate A	1	1	2
Employee Candidate B	2	2	1
Employee Candidate C	3	4	3
Employee Candidate D	4	3	4
Employee Candidate E	5	6	6
Employee Candidate F	6	5	5
Employee Candidate G	7	8	7
Employee Candidate H	8	7	8

Source : (Research Result, 2025)

Correlation analysis is conducted to measure the level of relationship between the ranking results obtained from the MAUT and MAUT-A methods. The aim is to determine the extent of consistency between the two methods in establishing the order of priority for alternatives. In this analysis, the rank correlation coefficient (Spearman's Rank Correlation) is used to indicate whether the two methods produce similar or significantly different rankings. The results of the rank correlation analysis are displayed in Figure 4.



Source : (Research Result, 2025)

Figure 4. Correlation Value

The results of the comparison of correlation values produced by the MAUT method and the MAUT-A method against the reference ranking. Based on the graph, the MAUT method produces a correlation value of 0.9688, while the MAUT-A method produces a higher correlation value of 0.9792. This difference indicates that the MAUT-A method has a better level of alignment with the comparative data or actual rankings, and also shows greater stability and reliability in handling data that may have asymmetric distributions. Thus, MAUT-A proves to provide superior performance in maintaining the consistency of decision results with actual preferences or evaluation results from the company.

The comparison between MAUT and MAUT-A shows fundamental differences in terms of the accuracy and robustness of decision-making results. Conventional MAUT is relatively simple, but it requires weights from an external weighting method to generate aggregate values, meaning the quality of the results is greatly influenced by the accuracy of the weighting method used. Other limitations include a tendency to produce uniform weight distributions and a reduced ability to capture significant variations between criteria, as well as a weakness in distinguishing alternatives when aggregate values are too close. Conversely, MAUT-A, through the application of nonlinear utility functions and deviation-based adjustment mechanisms, is able to generate weights that are more proportional to data variations directly, without relying entirely on external weighting methods. This gives MAUT-A a clearer discriminative power among alternatives. Furthermore, sensitivity tests show that the rankings produced by MAUT-A are more stable when criterion weights are modified, compared to MAUT, which is more susceptible to fluctuations. Thus, analytically, MAUT-A not only enhances assessment accuracy but also strengthens the consistency and reliability of decision-making results.

CONCLUSION

The reformulation of normalization within the framework of MAUT offers a more adaptive and accurate approach in handling asymmetric data in the MADM process. This new approach is capable of addressing the weaknesses of conventional normalization techniques that are often insensitive to skewed or extreme data distributions. By considering the asymmetry of the data, this method provides a more realistic representation of decision-makers' preferences, as well as enhances

the validity of the alternative ranking results. The implementation of this approach also demonstrates better stability in complex situations with high data heterogeneity, making it a significant contribution to the theoretical and practical refinement of MAUT methods. The comparison results of the correlation values produced by the MAUT method and the MAUT-A method against the reference ranking show that the MAUT method produces a correlation value of 0.9688, while the MAUT-A method yields a higher correlation value of 0.9792. This difference indicates that the MAUT-A method has a better level of conformity to the comparative data or actual ranking, and also demonstrates higher stability and reliability in handling data that may have an asymmetric distribution.

The MAUT-A method provides a theoretical contribution by introducing a renormalization mechanism and the utilization of standard deviation in generating criterion weights, so that each criterion is not only treated equally but also assessed based on the level of variation and its discriminative ability toward alternatives. This approach strengthens the theoretical foundation of MAUT by providing a more objective mathematical justification in the weighting process. From a practical relevance perspective, MAUT-A can assist decision-makers in various fields such as recruitment, supplier selection, or performance evaluation because the generated weights are more adaptive to data distribution and can more accurately represent real-world conditions. Nevertheless, its limitations lie in high computational complexity and the need for broader empirical testing to ensure result consistency across different decision-making contexts.

ACKNOWLEDGMENT

We extend our gratitude to the Ministry of Education, Culture, Research, and Technology of the Republic of Indonesia with contract numbers 123/C3/DT.05.00/PL/2025, 178/LL2/DT.05.00/PL/2025, and 005/UTI/LPPMI/E.1.1/VI/2025 for their support in funding this research.

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