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PERFORMANCE EVALUATION OF NEWTON–KONTOROVICH AND ADAPTIVE NEWTON LINE SEARCH ON MULTIVARIATE NONLINEAR SYSTEMS

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Abstract— *Solving multivariate nonlinear systems is essential in engineering, physics, and applied sciences. This study compares the performance of two numerical methods—Newton–Kontorovich and Interactive Newton–Raphson with Line Search—on trigonometric and exponential nonlinear systems. The methods are evaluated based on convergence rate, accuracy, and iteration efficiency through numerical simulations using MATLAB. The Newton–Kontorovich method, typically used for integral or differential equations, is compared with the adaptive line search strategy that enhances global convergence. Results show that the Interactive Newton–Raphson method achieves a smaller final error (5.95×10^{-2}) with stable convergence, while Newton–Kontorovich converges in fewer iterations but with larger error (3.126). These findings highlight the superiority of adaptive strategies for complex nonlinear systems. Practical implications include improved numerical reliability for applications in structural engineering, optimization, and scientific modeling.*

Keywords: *Interactive Newton Line Search, Newton Kontorovich, Numerical methods, Numerical Simulation.*

Intisari—*Pemecahan sistem nonlinier multivariat sangat penting dalam bidang teknik, fisika, dan ilmu terapan. Studi ini membandingkan kinerja dua metode numerik—Newton–Kontorovich dan Interactive Newton–Raphson dengan Pencarian Garis—pada sistem nonlinier trigonometrik dan eksponensial. Metode-metode tersebut dievaluasi berdasarkan laju konvergensi, akurasi, dan efisiensi iterasi melalui simulasi numerik menggunakan MATLAB. Metode Newton–Kontorovich, yang umumnya digunakan untuk persamaan integral atau diferensial, dibandingkan dengan strategi pencarian garis adaptif yang meningkatkan konvergensi global. Hasil menunjukkan bahwa metode Interactive Newton–Raphson mencapai kesalahan akhir yang lebih kecil ($5{,}95\times10^{-2}$) dengan konvergensi stabil, sementara Newton–Kontorovich konvergen dalam jumlah iterasi yang lebih sedikit tetapi dengan kesalahan yang lebih besar (3,126). Temuan ini menyoroti keunggulan strategi adaptif untuk sistem nonlinier kompleks. Implikasi praktis meliputi peningkatan keandalan numerik untuk aplikasi dalam rekayasa struktural, optimasi, dan pemodelan ilmiah.*

Kata Kunci: *Pencarian Garis Newton Interaktif, Newton Kontorovich, Metode Numerik, Simulasi Numerik.*

INTRODUCTION

Solving nonlinear equations has become a critical component of applied mathematics because of their central role in describing complex real-world phenomena [1]. These equations frequently appear in structural engineering, fluid dynamics, chemical reaction systems, economics, and biological modeling, where exact analytical solutions are often impossible to obtain [2]. For example, nonlinear systems are used to model structural deformation, analyze turbulent fluid flow, optimize engineering designs, and describe dynamic interactions in biological and economic systems. [3]. Therefore, numerical methods play a vital role as an effective alternative for finding approximate solutions that are accurate, efficient, and reliable [4]. This highlights the urgency of selecting appropriate numerical methods to ensure stable convergence and support various real-world applications in science and engineering.

The Newton-Kantorovich method generalizes the Newton method to Banach spaces and is often used to solve nonlinear integral or differential equations [5]. This method is a generalization of the classical Newton method that works in function space, but can be practically applied to discrete systems [6]. The efficiency of this method was tested by solving several examples whose exact solutions were already known [7]. This method uses repeated local linear approximations, which can improve numerical stability compared to the pure Newton method [8]. However, as a classical approach, the Newton–Kantorovich method also has several limitations. It is sensitive to the choice of initial values, may fail to converge for highly nonlinear or ill-conditioned systems, and often requires exact or well approximated derivatives. Moreover, its performance can deteriorate when applied to high-dimensional or complex multivariate problems, where convergence speed and accuracy are strongly influenced by the structure of the equations. These limitations motivate the use of more adaptive strategies that can handle a wider range of nonlinear problems effectively.

The Interactive Newton Line Search method is an iterative numerical approach commonly applied to solve nonlinear systems or optimization problems [9]. Unlike the classical Newton–Kantorovich method, which relies on fixed step updates and is often sensitive to initial guesses, this adaptive variant integrates a line search strategy to dynamically adjust the step size [10]. This modification aims to improve global convergence and stability while retaining the fast local convergence of the Newton–Raphson method [11]. By controlling the step length during iterations, the method can avoid divergence and achieve reliable convergence even from less ideal starting points [12]. As a result, line search–based Newton methods have been widely used in applied mathematics, computational physics, and engineering due to their balance between efficiency and robustness [13].

A number of previous studies have shown significant contributions in comparing the Newton–Kantorovich method by [6], [14]. Aghili, [6] employed Mellin and Kontorovich–Lebedev transformations to solve specific integral problems efficiently, demonstrating the effectiveness of the classical Newton–Kantorovich framework for structured mathematical functions. Kuryliak [14] focused on semi-local convergence analysis, highlighting improvements in the convergence behavior of Newton–Kantorovich iterations under certain conditions. While these studies provide valuable theoretical and methodological insights, they mainly concentrate on specific problem structures and do not address broader performance evaluations. In particular, there is limited research that systematically compares Newton–Kantorovich with adaptive Newton variants, such as line search methods, especially in the context of multivariate nonlinear systems involving trigonometric and exponential functions. This gap underlines the need for a comparative analysis that examines convergence rate, stability, and accuracy across different nonlinear problem types.

Several researchers are also associated with the Newton-Kontorovich method by [5], [15], [16]. Muslimov, [5] has carried out developments, including the Newton-Kantorovich method, which generalizes the Newton method to Banach spaces and is often used to solve nonlinear integral or differential equations. Regmi et al, [15] His research states that Newton Kontorovich's numerical results show that these new results can be used to solve nonlinear equations, but not for previous results. Jebreen et al, [16] This research aims to improve the computational efficiency of solving nonlinear algebraic problems with simple roots. To achieve this goal, a multi-step solver based on the Newton method is used. Building on these foundations, this study aims to conduct a systematic comparative analysis between the classical Newton–Kantorovich method and the adaptive Newton Line Search method. The main objective is to evaluate their performance on multivariate nonlinear systems involving trigonometric and exponential functions, focusing on convergence rate, numerical stability, and error accuracy. The contribution of this

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research lies in providing empirical evidence on the strengths and weaknesses of classical versus adaptive Newton methods, thereby filling the existing gap in the literature and offering insights for selecting suitable numerical techniques in practical applications.

Meanwhile, several other studies have also examined the Newton Interactive Line Search method with [9], [17], [18]. Lee et al, [9] explains that they modified the classical Newton method with line search. The results show that the back-mapping algorithm significantly improves accuracy and convergence speed when the proposed Line Search method is integrated. Abreu et al, [18] This algorithm is proposed based on the backward Euler method, which aims for accuracy and stability, and the Newton–Raphson method for solving unconstrained optimization problems. In this scenario, a line search strategy is applied to improve the convergence characteristics of the algorithm. The results show how line search and sub stepping strategies can improve the robustness of nonlinear analysis. Kirani et al, [17] Adapting the cubic regularization method into a line search framework to improve convergence efficiency and stability.

Previous studies have evaluated each method separately, but studies that directly compare the performance of both methods in the context of multivariate nonlinear systems, taking into account initial values and linear solution strategies, are still very limited. This study aims to systematically compare the performance of the Newton-Kantorovich and Newton-Raphson methods with Line Search in solving various types of representative nonlinear equations. The evaluation is based on numerical simulations to determine convergence speed, solution stability, and final absolute error. The results of this study are expected to provide a more accurate reference in selecting efficient and reliable numerical methods. Therefore, this research is expected to fill this gap and make a significant contribution in both theoretical and practical aspects, particularly in the selection of appropriate numerical methods and the development of mathematics learning and the application of numerical computation in the fields of science and engineering.

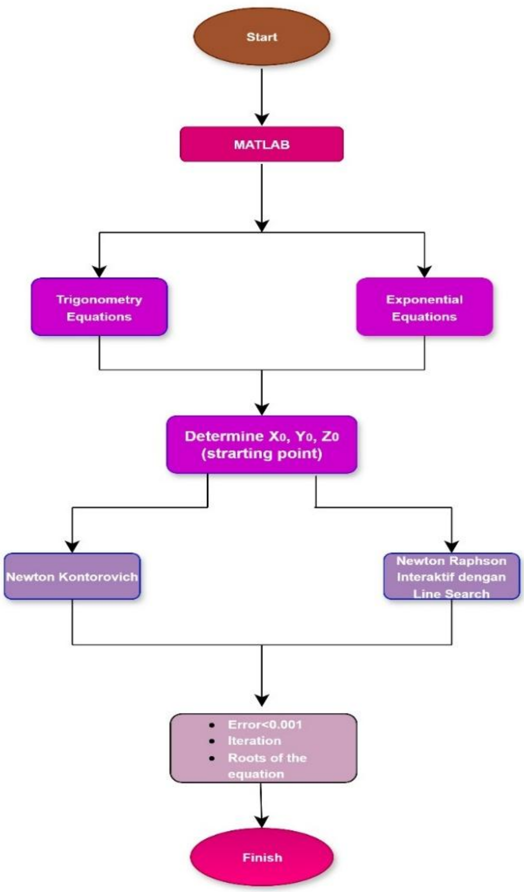
MATERIALS AND METHODS

This research is a quantitative study with a computational experimental approach, which aims to analyze and compare the performance of two numerical methods, namely Newton-Kantorovich and Interactive Newton Line Search, in solving non-linear equation systems. The study was conducted by directly simulating both methods using numerical programming software and analyzing the results based on convergence parameters, accuracy, and iteration efficiency. Both methods were chosen because they are known to provide efficient and accurate solutions through a systematic iterative process. The simulations were performed using MATLAB with an error tolerance limit of 0.001 and a maximum of 50 iterations. Each method will be tested against the three systems to assess their effectiveness in producing accurate and efficient solutions. This study uses three non-linear equation systems (samples (equation systems used for testing) conducted simultaneously in a single experiment, with variations in function complexity, number of variables, and degree of non-linearity to evaluate the performance and effectiveness of the combined methods applied. Furthermore, the questions used consist of:

$$f_1(x,y,z) = x^2 + y^2 + \tan(z) - 3 = 0$$

$$f_2(x,y,z) = e^x + y - z = 0$$

$$f_3(x,y,z) = x + y^3 - e^z = 0$$



Source: (Research Results, 2025)
Figure 1. Research Process

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Figure 1 illustrates the process of solving a system of nonlinear equations with three variables using a numerical solution approach with Matlab. The methods used are variants of Newton's method, namely the Newton Kontorovich method and the Interactive Line Search Newton method, which are commonly used in the fields of numerical analysis, optimization, and scientific computing. This research is applied to two types of problems, namely trigonometric and exponential, each of which is solved by determining the initial point (initial guess) x_0, y_0, dan z_0. Each problem was tested separately using the Newton Kontorovich method and the Interactive Newton Line Search method. The iteration process was automatically stopped when the resulting error value was less than 0.001, which is the convergence tolerance limit. The simulation data was analyzed to see the performance of both methods based on the number of iterations and the accuracy level of the solution.

RESULTS AND DISCUSSION

Result

In searching for solutions to non-linear equation systems, several numerical methods are used. Researchers use two numerical methods to solve these problems.

Newton Kontorovich Method

The Newton-Kantorovich method is an extension of the Newton-Raphson method used to solve nonlinear systems of equations and nonlinear differential equations in operator form, particularly when working in function space (functions as inputs/outputs, not just numbers) [19].. This method is very useful in solving complex problems such as partial differential equations (PDEs), continuous dynamic systems, and nonlinear physics and fluid simulations [20]. Here is the formula for the Newton-Cauchy method:

$$x_{k+1} = x_k - [F'(x_k)]^{-1} F(x_k) \tag{1}$$

Description:
x_k: solution approach in iteration to k
$F'(x_k)$: Fréchet derivative of F at x_k
$[F'(x_k)]^{-1}$: the inverse of the derivative
$F(x_k)$: function value at point x_k

Interactive Newton Method Line Search

The Newton-Raphson method with line search is an extension of the classical Newton-Raphson method for solving nonlinear systems of equations or minimizing functions [21]. In this approach, after the search direction Δx is obtained from the Newton method, the update step is not performed immediately, but is refined through an optimal step search (line search) to ensure more stable and rapid convergence [22]. Here is the formula for the Interactive Newton-Raphson Method with Line Search:

$$x_{k+1} = x_k + t_k \, . \, P_k \tag{2}$$

Description:
x_k: current solution (guess to k)
P_k: direction of movement (direction of correction)
t_k: step size adjusted through the line search process

This method is then used to solve non- linear equation systems. The method used in this study aims to solve non-linear equation systems (trigonometric and exponential). In this approach, a combination of several numerical methods is used, namely the Newton Kontorovich Method and the Interactive Newton Line Search Method.

This section presents the implementation and results of several numerical methods in solving nonlinear equation systems. The implementation was carried out using Matlab scripts for each method. The numerical results and solution graphs of each method were then compared. The researcher used three types of nonlinear equation systems, namely trigonometric and exponential equation systems. Each system is analyzed using a numerical approach based on the Newton method. The methods used include the Newton-Kantorovich method and the Newton-Raphson method with line search.

Table 1 presents the Matlab script for each numerical method used, namely Newton Kontorovich and Interactive Newton Line Search. This script represents the algorithm used in solving nonlinear problems. To test the performance of the methods, the researcher conducted two experiments on each type of trigonometric and exponential problem, with an error tolerance value of 0.001 and a maximum iteration limit of 50.

Table 1. Newton Kontorovich and Interactive Newton Line Search numerical method script

Method	Script

Newton Kontorovich	iter = 1:imax F = f_func(x); J = J_func(x);
	dx = -J \ F; x_new = x + dx;
	error = norm(dx, inf);
	fprintf('%d\t\t%.6f\t%.6f\t%.6f\t%.6f\n', iter, x(1), x(2), x(3), error);
	ror < tol break;
	end
	x = x_new;

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Method	Script

Newton Interactive Line Search	for k = 1:imax
	  Fval = [f1(x); f2(x); f3(x)];
	  normF = norm(Fval);
	
	  if normF < tol
	    break;
	  end
	
	  h = 1e-6;
	  J = zeros(3, 3);
	  for i = 1:3
	    dx = zeros(3, 1);
	    dx(i) = h;
	    f_perturb = [f1(x + dx); f2(x + dx); f3(x + dx)];
	    J(:, i) = (f_perturb - Fval) / h;
	  end

Source: (Research Results, 2025)

The script in Table 1 above, namely the Newton Kontorovich method script and the Interactive Newton Raphson with Line search script, is an iterative implementation of the Newton Kontorovich method and the Interactive Newton Raphson with Line search method to find the roots of non-linear equations by utilizing the script to perform simulations.

Table 2. Simulation Results

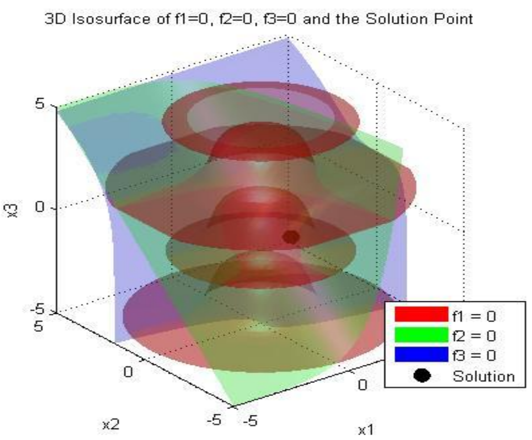
No	Metode	Iteration	X	Y	Z	Error
1	Newton Kontorovich	8	0.635	−0.863	−0.626	3.126*e*
2	Newton Interactive Line search	50	−1.650	1.959	1.847	0.595*e* − 001

Source: (Research Results, 2025)

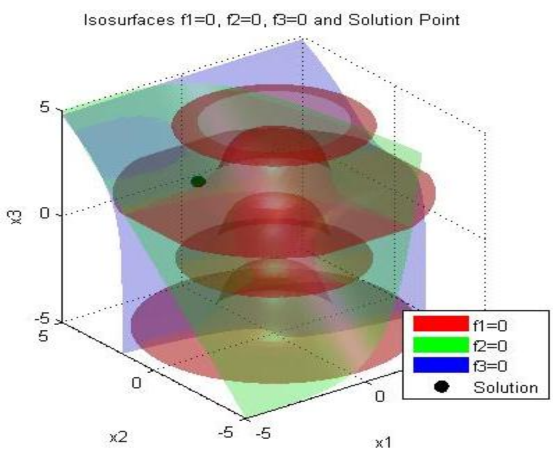
After performing simulations using both methods on a system of three nonlinear equations, the computational results showed that the Newton-Kontorovich method required 8 iterations to approximate the solution. The simulation results showed values of x = 0.635, y = -0.863, and z = -0.626, with an error rate of 3.126. Although the simulation was carried out up to the maximum number of iterations, the resulting error was still quite large, indicating that this method is less effective in achieving convergence in the tested system of equations.

Newton Interactive Line Search also requires 50 iterations, although 50 iterations provide much more efficient and accurate results. The solution values obtained from this simulation are x= -1.650, y= 1.959, and z= 1.847, with an error rate of 0.595×10^{-1}, indicating that this method successfully finds a solution that is very close to the actual root of the tested system of equations.

In this case, the starting point taken was [0.4,1.6,1.5]. The difference in performance between the methods became more apparent. The Interactive Newton Raphson method with line search showed the best performance, with a very small error of 0.595e-001 and 50 iterations, indicating that this approach was the most accurate and efficient for this case. In contrast, the Newton-Kontorovich method exhibited convergence delays, despite having 8 fewer iterations than the Newton method, the error achieved was still as high as 3.126. The researchers provide a visual representation of the computational results in Table 2, presented in three-dimensional graphs in Figures 2 and 3. These graphs illustrate the interaction between the numerical solution and the functional field of the nonlinear system of equations that was solved.



Source: (Research Results,2025)
Figure 2. Showing the solution to the problem using the Newton-Cauchy method



Source: (Research Results,2025)
Figure 3. Showing the solution to the problem using the Newton Interactive Line Search Method

DISCUSSION

The results show that the Interactive Newton Raphson method with Line Search has the best performance, especially in the case of non-linear equation systems, with the smallest error of $0.595e-001$ and a more efficient number of iterations. This study is superior to several previous studies that have discussed the application of numerical methods in solving non-linear equation systems. Penelitian oleh Mumtazi et al, [23] It was found that the Secant method is more efficient for finding approximate values of polynomial functions, exponential functions, trigonometric functions, and mixed functions with a range of 0.1% - 0.5%. For polynomial functions, the Secant method has an accuracy of 30%, for exponential functions the Secant method has an accuracy of 28%, and for trigonometric functions the Secant method has an accuracy of 97% compared to the Steffensen method. Penelitian yang dilakukan oleh Rahmatullah, [24] The results show that, in terms of the number of iterations, the Newton-Raphson method outperforms the Steffensen method for polynomial and trigonometric functions by 84% and 62%, respectively, while for exponential functions, the Steffensen method outperforms by 12%.

Research conducted by Kirani et al, [17] The convergence speed of determining the root of a nonlinear equation or function using the Fixed Point method with the Newton Raphson method using Matlab. The calculation results using the Matlab application program show that the convergence speed towards the root using the Newton-Raphson method is faster than the Fixed Point method. The average iteration speed with an error rate of 0.001 is 64% faster. Meanwhile, the results of research conducted by Silviana Dewi Anastasya, [25] shows that the Newton-Raphson method converges faster than the Fixed-Point method, with an average of 64% fewer iterations to reach an error tolerance value of 0.001. The researchers have several advantages over previous studies. The Interactive Newton Line Search method used is an innovation of the classical method, with faster and more stable convergence capabilities, especially in multivariate non-linear equation systems. Evaluation based on error and number of iterations provides a more comprehensive picture, while the use of trigonometric and exponential functions demonstrates the strength of the method in more complex cases. However, this study still has weaknesses, such as limited method comparisons, no testing on systems or roots that are the same as previous studies, and no analysis of computation time and method stability for initial guesses that are far from the root. Nevertheless, this study still makes an important contribution to the development of more efficient and adaptive numerical methods.

This study is limited to low-dimensional systems (three variables) and focuses on two representative Newton-based approaches. Further research should expand the comparison to include the Broyden, Secant, and Trust Region methods for high-dimensional problems, with detailed analysis of computational complexity, memory usage, and robustness to poor initial guesses.

CONCLUSION

The results of the numerical simulation show that the Interactive Newton-Raphson method with Line Search demonstrates the best performance by producing the solution closest to the actual root with an error of 0.595×10^{-1} and has superior convergence stability and accuracy compared to the Newton-Kontorovich method, even with the same number of iterations. Thus, this method is considered the most effective in solving the nonlinear equation system tested. For further research, it is recommended that the Interactive Newton-Raphson method with Line Search be tested on more complex and high-dimensional nonlinear equation systems to evaluate its performance and scalability more comprehensively. The application of this method to real-world case studies in engineering, economics, and applied sciences will strengthen its practical validity. Finally, the development of interactive visualization tools based on MATLAB will be highly useful in assisting numerical analysis and can be utilized in the learning and training of algorithms in a more intuitive manner.

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