

COMPARATIVE ANALYSIS OF COGNITIVE DIAGNOSTIC MODELS IN POMDP-BASED ADAPTIVE TESTING

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Abstract—Recent progress in learning analytics and educational data mining has accelerated the development of adaptive learning systems, where Cognitive Diagnostic Computerized Adaptive Testing (CD-CAT) has emerged as a significant approach. CD-CAT employs Cognitive Diagnosis Models (CDMs) to generate detailed evaluations of student competencies. Nevertheless, increasing numbers of attributes and test items introduce challenges related to the complexity and uncertainty of adaptive policies. This research investigates and compares four cognitive diagnosis models, namely GD-DINA, MIRT, MCD, and KaNCD, within an adaptive testing framework based on a Partially Observable Markov Decision Process (POMDP). The evaluation was conducted using the ASSISTments dataset with Accuracy, AUC, and expected reward as performance metrics. The findings indicate that KaNCD achieved the best overall performance, obtaining the highest diagnostic accuracy (0.7503) and AUC score (0.7410), while also maintaining stable results in POMDP-based adaptive testing (expected reward = 0.792). Although GD-DINA produced the highest expected reward (0.901), its accuracy was comparatively lower. Meanwhile, MCD demonstrated a balanced performance, with high accuracy (0.7464) and strong adaptability of policy (reward = 0.873). Overall, these results suggest that KaNCD offers the most effective balance among accuracy, interpretability, and efficiency in POMDP-based adaptive testing systems.

Keywords: Computerized Adaptive Testing, Cognitive Diagnostic Model, KaNCD, MCD, POMDP.

Intisari—Perkembangan analitik pembelajaran dan penambangan data pendidikan telah mempercepat pengembangan sistem pembelajaran adaptif, di mana Cognitive Diagnostic Computerized Adaptive Testing (CD-CAT) menjadi salah satu pendekatan yang menonjol. CD-CAT menggunakan Cognitive Diagnosis Model (CDM) untuk menghasilkan diagnosis yang rinci terhadap kemampuan siswa. Namun demikian, bertambahnya jumlah atribut dan item menimbulkan tantangan dalam pengelolaan kompleksitas serta ketidakpastian kebijakan adaptif. Penelitian ini menganalisis dan membandingkan empat model diagnosis kognitif, yaitu GD-DINA, MIRT, MCD, dan KaNCD, pada sistem pengujian adaptif berbasis Partially Observable Markov Decision Process (POMDP). Evaluasi dilakukan menggunakan dataset ASSISTments dengan metrik Akurasi, AUC, dan expected reward. Hasil penelitian menunjukkan bahwa KaNCD memberikan performa terbaik secara keseluruhan dengan memperoleh akurasi diagnostik tertinggi (0.7503) dan nilai AUC terbesar (0.7410), serta mempertahankan performa yang stabil pada pengujian adaptif berbasis POMDP (expected reward = 0.792). Meskipun GD-DINA menghasilkan expected reward tertinggi (0.901), tingkat akurasinya relatif lebih rendah. Di sisi lain, MCD memperlihatkan performa yang seimbang melalui akurasi tinggi (0.7464) dan kemampuan adaptasi kebijakan yang kuat (reward = 0.873). Secara keseluruhan, temuan ini menunjukkan bahwa KaNCD mampu memberikan keseimbangan terbaik antara akurasi, interpretabilitas, dan efisiensi dalam sistem pengujian adaptif berbasis POMDP.

Kata Kunci: Ujian Adaptif Berbasis Komputer, Diagnosis Kognitif Model, KaNCD, MCD, POMDP.

INTRODUCTION

The advances in learning analytics and educational data mining have enabled the development of adaptive and personalized assessment systems [1], [2]. Computerized Adaptive Testing (CAT) plays a central role in this progress by selecting items dynamically based on student responses. Recent studies show that machine learning can further enhance CAT through improved item selection, uncertainty modeling, and integration with cognitive diagnostic frameworks [3]. However, traditional Item Response Theory (IRT)-based CAT remains limited, as IRT is unidimensional and unable to capture the nuanced cognitive attributes required for diagnostic feedback [4], [5].

To address these limitations, Cognitive Diagnosis Models (CDMs) were introduced to estimate attribute-level mastery using the Q-matrix framework [4], [5]. CD-CAT systems built on CDM provide more interpretable and actionable diagnostic information than traditional IRT-based CAT. Their performance can be further improved through maximum information gain or mutual information strategies [3]. Recent heuristic item-selection rules also enhance efficiency and ensure balanced item exposure [6], [7].

Despite substantial progress in CD-CAT, uncertainty remains a major challenge because noisy attribute mastery and response errors make item selection highly sensitive to belief updates. Empirical evidence indicates that early incorrect responses often mislead the item selection logic, causing the test to shift toward suboptimal difficulty levels demonstrating how uncertainty shapes adaptive decision-making [3], [6]. Since adaptive testing operates under partial observability, optimal item selection must be probabilistic [8]. This motivates the use of POMDP, which models uncertainty explicitly and optimizes sequential decisions through belief-state policies [3], [9].

Compared to traditional CAT, POMDP explicitly incorporates uncertainty in state estimation, enabling more robust policy decisions in the presence of response noise, attribute correlations, or nonlinear interactions between items and attribute conditions commonly observed in CD-CAT settings [3], [6], [10]. Reinforcement learning research further supports POMDP and policy-gradient approaches for complex partially observable testing environments [8], [11].

Recent CDM research has introduced more advanced model architectures. NeuralCD improves diagnostic accuracy by modeling non-linear interactions under monotonicity constraints [8],

[11]. KaNCD applies the Kolmogorov–Arnold representation to achieve high accuracy with fewer parameters [9]. GD-DINA enhances the classical DINA model through gradient-based optimization [12], and MIRT extends IRT to multidimensional traits [4], [13]. Each model presents trade-offs: GD-DINA is interpretable but initialization-sensitive; MIRT captures latent ability correlations but lacks attribute-level clarity; MCD learns complex non-linear patterns but sacrifices interpretability; KaNCD is accurate and efficient, though its adaptive-testing performance remains underexplored.

Although prior work has explored CDMs and adaptive testing independently, no earlier study has conducted a unified comparison of GD-DINA, MIRT, MCD, and KaNCD within a single POMDP-driven adaptive testing framework. Earlier research typically evaluates CDMs in static settings [5], [8], [11], compares item-selection rules without varying the diagnostic model [6], or applies reinforcement learning to CAT without integrating heterogeneous model families [3], [10], [14]. This leaves a critical gap, as these four models represent fundamentally different methodological paradigms—latent-trait (MIRT), diagnostic-classification (GD-DINA), and neural architectures (MCD, KaNCD)—which can substantially influence belief updates and reward optimization in POMDP-based systems.

To fill this gap, the present study provides the first systematic evaluation of all four models within a unified adaptive testing pipeline, examining both (1) diagnostic performance (AUC and accuracy) and (2) POMDP policy effectiveness through expected reward. This dual-perspective evaluation follows recommendations in CD-CAT and reinforcement learning literature, emphasizing the need to assess diagnostic quality and adaptive policy performance together [3], [6], [10], [14]. Based on the theoretical and empirical gaps identified above, this study addresses the following research questions:

RQ1: Which CD model produces the most stable and accurate belief updates when integrated into a POMDP-based adaptive testing system?
RQ2: Which CD model provides the optimal balance between accuracy, interpretability, and policy stability under uncertainty?

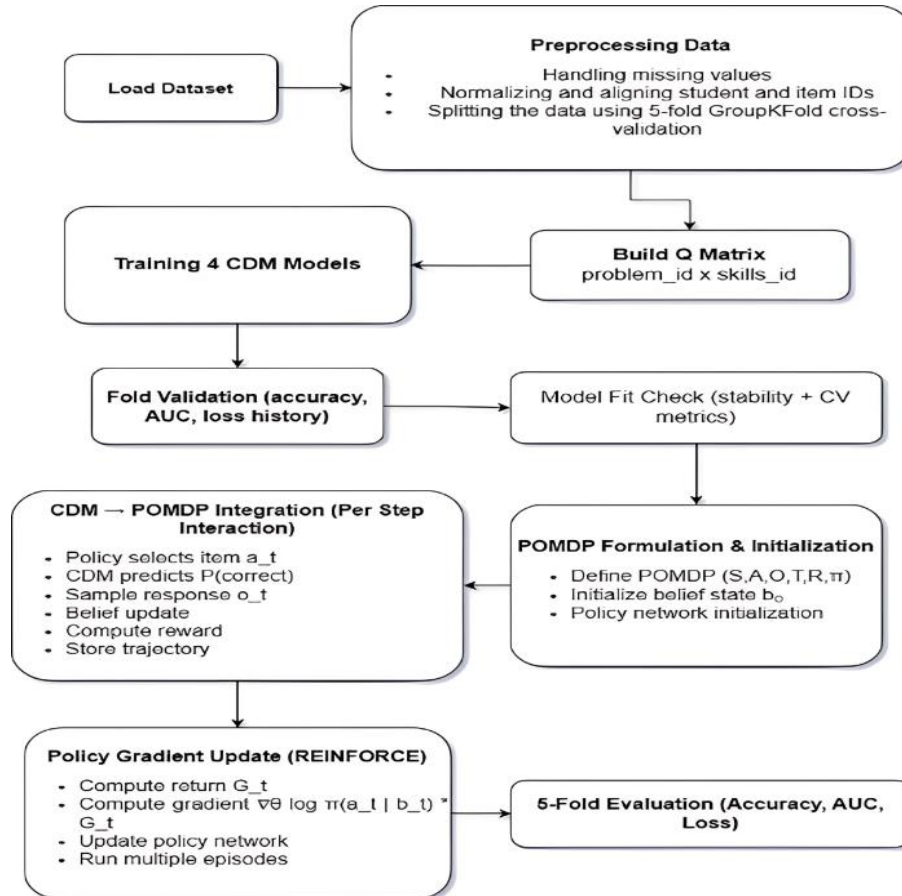
In summary, this study fills the gap by integrating GD-DINA, MIRT, MCD, and KaNCD into a unified POMDP-driven CD-CAT pipeline and evaluating them using diagnostic accuracy, AUC, and expected reward. The goal is to identify which model provides the most stable belief updates and the most effective policies under uncertainty [3], [5], [10], [12], [14], [15]



MATERIALS AND METHODS

This study went through several stages of systematic research methodology, starting from the pre-processing stage to testing the model's performance in an adaptive testing system based on

Partially Observable Markov Decision Process (POMDP) [3]. This study combines four cognitive diagnosis models, GD-DINA, MIRT, MCD, and KaNCD, as student ability estimators, with the POMDP approach as the test system's adaptivity controller.



Source: (Research Results, 2025)

Figure 1. Research Flow

Figure 1 shows the research flow used in this study, starting from data preprocessing, CDM training, and POMDP integration, up to policy optimization and final evaluation.

Data Preparation and Pre-processing

The dataset used in this study comes from [ASSISTmentsData - 2009-2010 ASSISTment Data](#), an online learning platform widely employed in Adaptive Learning and Computerized Adaptive Testing (CAT) research [16]. The publicly available dataset contains 346,860 student interactions, involving 4,217 unique students, 26,688 problem items, and 150 skill attributes (*skill_id*) across various mathematics topics. Key features include *user_id*(unique identifier for each student), *problem_id*(item identifier), *skill_id*(cognitive skill tag), *correct* (correctness of the response (binary)),

ms_first_response(response time in milliseconds), *attempt_count*(number of attempts), *hint_count* (number of hints requested), *answer_type* (response format category).

Since multiple cognitive skills may be associated with a single item, the dataset naturally supports multi-attribute diagnosis, making it suitable for CDM-based modeling. Data Preprocessing was performed to improve the stability of gradient-based models [17]:

- Missing Value Cleaning: Records with missing *skill_id* or *correct* values were removed.
- Outlier Handling: Response times (*ms_first_response*) exceeding three standard deviations (assumed $|Z| > 3$) were discarded as outliers.
- Feature Normalization: Continuous behavioral variables (e.g., *attempt_count* and *hint_count*)



were standardized using Z-score normalization to stabilize gradient-based optimization [14].

1. Feature Encoding and Normalization

Categorical variables were encoded into numeric form, while continuous behavioral variables (attempt_count, hint_count) were normalized using Z-score. This step is necessary to stabilize gradient-based optimization and prevent certain features from dominating the learning process [18].

2. Handling Data Imbalance and Psychometric Validity

The distribution of correct and incorrect responses in educational datasets inherently reflects item difficulty and student ability distribution. Therefore, data imbalance was not addressed using conventional oversampling or undersampling techniques. Such interventions could bias item parameter estimation and compromise the psychometric validity of the diagnostic model, as the modified data would no longer accurately represent actual student-item interactions [19]. This natural imbalance is instead modeled by the CDM to estimate parameters accurately.

3. Cross-Validation and Data Splitting

To ensure stable and independent evaluation results and prevent data leakage [3], five-fold cross-validation (GroupKFold) was applied. Data splitting was performed based on user_id (groups=df["user_id"]), ensuring that all interactions (rows) from the same student appear in only one fold (train or test). This method effectively eliminates the risk of leakage common in sequential data. Model performance is reported as the average across all folds.

Q-Matrix Construction

In cognitive diagnostic modeling, the Q-matrix is a binary incidence matrix that encodes the relationship between items and cognitive attributes. Each row represents an item, and each column represents a cognitive skill attribute. A value of 1 indicates that the attribute is required to answer the item correctly, whereas 0 indicates no direct involvement. Formally, the Q-matrix is defined as:

$$Q = [q_{jk}] \in \{0,1\}^{J \times K} \quad (1)$$

From the equation (1), J denotes the number of items, K the number of cognitive attributes, and

$q_{jk} = 1$ if item j requires attribute k , and $q_{jk} = 0$ otherwise [20], [21]. In the ASSISTments dataset, the Q-matrix is constructed directly from the skill_id column. Since a single item (problem_id) can be tagged with multiple skill_ids (as observed in the raw data), Q is naturally encoded as a multi-hot matrix, meaning that a single item row can activate multiple attribute columns simultaneously.

Cognitive Diagnostic Model

The four cognitive diagnostic models compared are GD-DINA, MIRT, MCD, and KaNCD. Each represents a different paradigm, from deterministic and latent trait to neural interpretive.

1. GD DINA Model

The GD-DINA model is a variant of the Deterministic Input Noisy AND gate (DINA) that utilizes the gradient descent algorithm to accelerate parameter convergence [17] compared to the Expectation-Maximization approach.

$$P(X_{ij} = 1 | \alpha_i, q_j) = (1 - s_j)^{\eta_{ij}} g_j^{(1-\eta_{ij})} \quad (2)$$

The equation (2) is the probability of student i answering item j correctly, with s_j as the slip parameter, g_j as the guess parameter, and $\eta_{ij} = 1$ if all attributes in q_j have been mastered by student [20], [21]. In this study, the model input consists of the student ID, item ID, and the corresponding row of the Q-matrix representing the attributes required by the item. The hidden representation is designed to match the number of attributes (150 dimensions) to ensure the model is sensitive to the cognitive structure encoded in the Q-matrix. Training is performed using a five-fold GroupKFold scheme to prevent response leakage across students, while a batch size of 64 and a small learning rate are employed to maintain the stability of parameter updates, which are particularly sensitive in DINA-based models. Table 1 below summarizes the training configuration used in this study, including model dimensions, optimization setup, and cross-validation settings

Table 1. DINA Training Configuration

Parameter	Value
Dimensi hidden	150
Batch size	64
Optimizer	Adam
Learning rate	0.002
Epoch	20
Cross-validation	GroupKFold (5 folds)
Loss	Binary Cross-Entropy

Source: (Research Results, 2025)



2. MIRT (Multidimensional Item Response Theory)

The MIRT model extends IRT to a multidimensional space to represent various aspects of student ability. The probability that student i will answer item j correctly is given by:

$$P(X_{ij} = 1) = \frac{1}{1 + \exp[-(a_j^T \theta_i + b_j)]} \quad (3)$$

with a_j as the discrimination vector and b_j as the item difficulty parameter [5], [7]. This approach allows for the simultaneous estimation of abilities across multiple dimensions, MIRT can estimate both item parameters and student abilities concurrently, enabling information to be “borrowed” across dimensions via the covariance of latent abilities. In this study, MIRT is not only employed as a theoretical foundation but also operationalized as a latent variable model to produce multidimensional ability estimates and item characteristic parameters. The model is trained using five latent dimensions, chosen to maintain estimation stability while preserving the representation of student ability variation. Training is conducted using a five-fold GroupKFold scheme to ensure that no student appears in both the training and testing sets simultaneously. Optimization is performed with Adam using a learning rate of 0.001, a dropout rate of 0.3, a mini-batch size of 512, and a total of 30 epochs. The resulting trained parameters— θ_i , a_j , and b_j —are then extracted and used as the basis for calculating response probabilities in the subsequent adaptive modeling phase.

Table 2. MIRT Training Configuration

Parameter	Value
Dimensi laten	5
Batch size	512
Optimizer	Adam
Learning rate	0.001
Epoch	30
Cross-validation	KFold (5 folds)
Loss	Binary Cross-Entropy

Source: (Research Results, 2025)

3. MCD (Multidimensional Cognitive Diagnosis)

The MCD model integrates the explicit attribute structure of CDMs with the flexibility of multidimensional latent models like MIRT [3], yielding smoother probabilistic estimates compared to deterministic models such as DINA. Its response function leverages a non-linear combination of attributes and item parameters, enabling the model to capture multidimensional interactions without losing the core diagnostic structure. In this study, MCD is trained with a 50-dimensional latent embedding to represent a richer

ability structure, combined with five-fold cross-validation to ensure stable estimation across variations in student response data. The use of small batches and Adam-based optimization helps maintain learning stability within a relatively large parameter space. The detail of training configuration is explained in table 3 below.

Table 3. MCD Training Configuration

Parameter	Value
Dimensi laten	50
Batch size	64
Optimizer	Adam
Learning rate	0.002
Epoch	10
Cross-validation	KFold (5 folds)
Loss	Binary Cross-Entropy

Source: (Research Results, 2025)

4. KaNCD (Kolmogorov–Arnold Neural Cognitive Diagnosis)

KaNCD (Kolmogorov–Arnold Neural Cognitive Diagnosis) is an extension of NeuralCD [9] that replaces the Multi-Layer Perceptron with Kolmogorov–Arnold Networks (KAN) to enhance interpretability and training efficiency [12]. The model is grounded in the Kolmogorov–Arnold Representation Theorem, which states that any continuous multivariate function can be represented as a superposition of univariate functions [15]. This principle allows KAN to capture non-linear relationships among cognitive attributes through simple, interpretable components, while maintaining monotonicity such that increases in attribute mastery do not reduce the probability of a correct response [12]. Modern KAN architectures are further supported by Li [13], who demonstrated that implementation can be reduced to radial basis networks, with the FastKAN variant accelerating training up to 3.3× without sacrificing accuracy.

In this study, KaNCD is implemented as a Q-matrix-based cognitive diagnostic model to predict response success probabilities based on the combination of student embeddings, item embeddings, and skill representations. The model is trained using a 50-dimensional representation, a batch size of 64, Adam optimizer with a learning rate of 0.002, and 10 epochs as described in table 4 below. Evaluation is performed using five-fold KFold cross-validation to ensure stable generalization, yielding an average AUC of 0.7503 and accuracy of 0.7410 across all folds. All model parameters including student embeddings, item embeddings, univariate KAN transformation functions, and sigmoid output parameters are learned end-to-end and stored per fold for full reproducibility.

Table 4. KaNCD Training Configuration

Parameter	Value
Dimensi embedding	50
Batch size	64
Optimizer	Adam
Learning rate	0.002
Epoch	10
Cross-validation	KFold (5 folds)
Loss	Binary Cross-Entropy

Source: (Research Results, 2025)

The KaNCD predictions are then used to construct the initial skill mastery estimates (initial belief state) for the subsequent POMDP modeling phase. This approach preserves the theoretical foundation of KAN while providing a transparent, efficient, and replicable operational implementation, consistent with the interpretability requirements of the cognitive diagnostic domain [14], [22].

Partially Observable Markov Decision Process (POMDP)

The integration of CDM models with POMDP is designed to enable adaptive item selection based on student ability estimates obtained from MIRT, KaNCD, MCD, and GD-DINA. This approach formalizes the assessment process as sequential decision-making under uncertainty, as described by Xiao Li et al. [23] and Xiang et al. [10], such that each item selection step maximizes diagnostic information while accounting for the uncertainty in student responses. As an initial verification step before integrating CDM models into the POMDP framework, all diagnostic models (MIRT, KaNCD, MCD, and GD-DINA) were first evaluated using commonly employed model fit indices in CDM studies. This evaluation included examining parameter accuracy, convergence stability in cross-validation, and the consistency of attribute mapping with student responses, as recommended in the modern CDM literature [24]. This analysis ensures that the parameters learned by each model adequately fit the cognitive structure of the data before being used as components for belief state estimation in the POMDP-based adaptive decision-making process. Consequently, the POMDP integration does not operate independently of the underlying diagnostic model quality but rather builds upon models that have been empirically validated.

1. POMDP Formula

A POMDP is defined as a tuple:

$$\mathcal{P} = (S, A, O, T, \Omega, R, \gamma) \quad (4)$$

The context of cognitive diagnosis from the Equation 4 is:

- a) State $s_t \in S$: the student's latent ability vector.
 - a. For MIRT: $\theta_i \in \mathbb{R}^d$.
 - b. For CDMs (MCD, KaNCD, GD-DINA): attribute mastery vector $\alpha_i \in [0,1]^K$.
- b) Action $a_t \in A$: the item selected by the system from the pool of all items.
- c) Observation $o_t \in O$: the student's response to the item, with binary values $o_t \in \{0,1\}$.
- d) Transition / belief update T : the system's update of its belief about the student's abilities.
- e) Reward R : the diagnostic information obtained from each student response.
- f) γ : the discount factor controlling the decision-making horizon.

2. Belief Update

Because the learner's latent ability cannot be observed directly, the system maintains a belief state b_t , which represents the probability distribution over possible ability states at step t . After the learner answers item a_t with observation o_t , the belief state is updated proportionally to the likelihood of observing o_t under each candidate state:

$$b_{t+1}(s') \propto P(o_t | s', a_t) b_t(s') \quad (5)$$

A normalization step is then applied so that all updated belief values sum to one. This Bayesian update mechanism allows the system to incorporate new evidence from each response and adjust its estimate of the student's ability over time.

3. Reward

The reward is designed to quantify the amount of new information gained from a student's response. Formally:

$$R(s_t, a_t, o_t) = I[o_t = 1] \quad (6)$$

In the implementation from equation (6), the reward is assigned 1 for a correct response and 0 for an incorrect response. This reward effectively reinforces item selection that maximizes the likelihood of obtaining informative responses and provides stability for policy gradient optimization, consistent with findings by Lin et al [8]. and Shakyu [11].

4. Policy Gradient

Adaptive item selection is controlled by a policy $\pi_\theta(a_t|b_t)$, implemented as a neural network that receives the belief state and outputs a

probability distribution over candidate items. The policy is optimized using the REINFORCE algorithm, which updates the parameters based on the log-probability of the selected action weighted by the cumulative reward. This approach allows the system to iteratively adjust its item-selection strategy toward actions that yield higher long-term rewards, following standard policy gradient methods in reinforcement learning.

5. Integration of CDM into POMDP

Using the above formulation, the integration of each model proceeds as follows:

- The CDM model (MIRT, MCD, KaNCD, GD-DINA) produces the probability of a correct response, as the modelled in equation (7):

$$P(o_t = 1 | s_t, a_t) \quad (7)$$

- This probability is used to sample simulated responses, as testing combines real interaction data with a simulation process (self-play).
- The simulated response is treated as the observation o_t .
- The belief state is updated according to the belief update function, reflecting the system's latest understanding of the student's ability.
- The policy gradient selects the next item based on the updated belief.

Through this mechanism, the adaptive system can optimize the item sequence to maximize diagnostic information, as recommended by Li [25], Gao [5], and Xin [7] for modern CAT-CDM frameworks.

Training and Evaluation Strategy

All models were trained using general parameters: learning rate between 0.001–0.01, batch size 32–128, and number of epochs 10–30 depending on model complexity. Optimization was performed using Adam for probabilistic models (MIRT, MCD, KaNCD) and Stochastic Gradient Descent for GD-DINA [15], [26]. Performance evaluation was conducted using 5-Fold Cross Validation, employing the Accuracy, AUC, and Loss History metrics as recommended by Liu. [3]. The final value was taken from the average between folds to ensure model stability against data variation [24].

RESULTS AND DISCUSSION

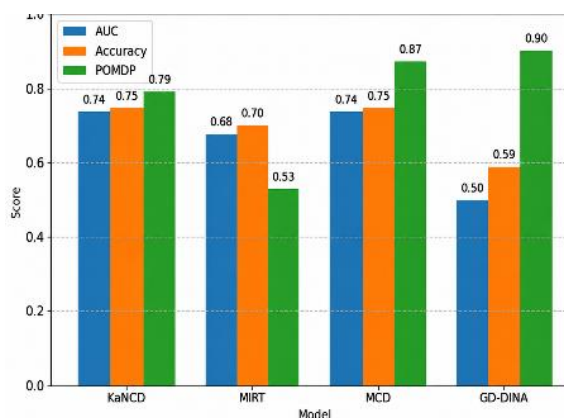
The experiment evaluated the performance of four cognitive diagnosis models GD-DINA, MIRT, MCD, and KaNCD within a POMDP-based adaptive testing system. Using 5-Fold Cross Validation, model performance was assessed through AUC,

accuracy, and expected POMDP reward. Gradient-descent optimization was applied to improve convergence stability, consistent with findings by Haji and Abdulazeez [17] while the use of Kolmogorov–Arnold Networks in KaNCD enabled efficient nonlinear function representation as supported by the Kolmogorov–Arnold theorem [15], [13]. The overall test results are shown in Table 5 and Figure 2. The values presented are the average results of five cross-validations for each model.

Table 5. Training Result

Model	AUC	ACC	POMDP ACC
KaNCD	0.7410	0.7503	0.792
MIRT	0.6768	0.6993	0.533
MCD	0.7373	0.7464	0.873
GD-DINA	0.4953	0.5874	0.901

Source: (Research Results, 2025)



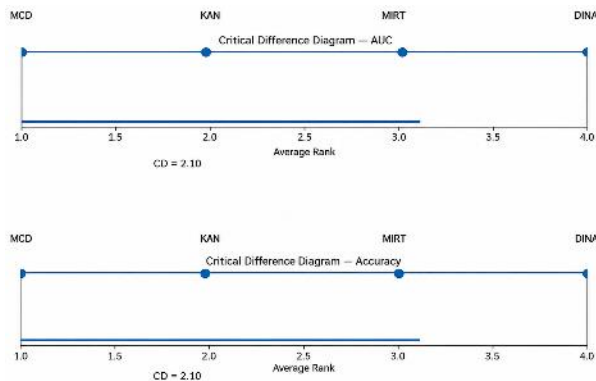
Source: (Research Results, 2025)

Figure 2. Model Comparison

Analysis of Model Training Results

To assess whether the four models differ significantly in performance, a Friedman test was conducted using the per-fold AUC and accuracy values from the 5-fold cross-validation. The results show significant differences for both AUC ($\chi^2 = 15.00$, $p = 0.0018$) and accuracy ($\chi^2 = 15.00$, $p = 0.0018$), indicating that the model rankings are unlikely due to chance.

A Nemenyi post-hoc test was then performed to analyze pairwise differences. The Critical Difference (CD) plots in Figure 3 illustrate the average model ranks and the CD threshold (CD = 2.10), highlighting which model pairs differ significantly—those separated by more than the CD line—and which fall within the non-significant interval.



Source: (Research Results, 2025)
Figure 3. Critical Difference Diagram

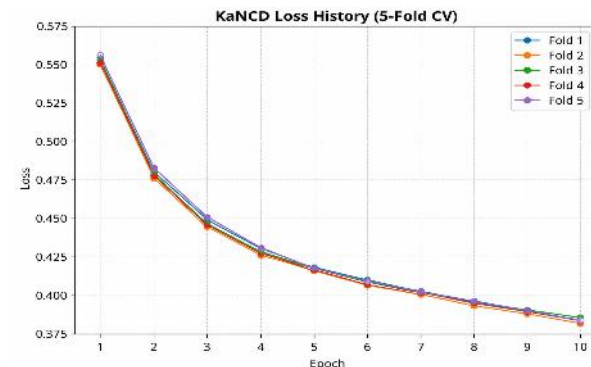
A. KaNCD

The KaNCD model achieves the highest performance among all evaluated models, with an AUC of 0.7410 and an accuracy of 0.7503. This superior performance reflects the ability of Kolmogorov–Arnold Networks (KAN) to model complex non-linear cognitive interactions using a significantly lower parameter count compared to traditional multilayer perceptrons [12], [13], [15]. Such parameter efficiency contributes to stable convergence and prevents overfitting in high-dimensional educational datasets. From a CDM perspective, KaNCD aligns well with the foundational idea that student responses arise from structured latent skills. The monotonicity constraint embedded in KaNCD ensures that increasing mastery of a cognitive attribute never decreases the probability of a correct response, consistent with the interpretability principles emphasized in cognitive diagnosis theory [12]. This makes KaNCD not only accurate but theoretically coherent with CDM assumptions.

The Kolmogorov–Arnold theorem provides the mathematical foundation for KAN, stating that any multivariate continuous function can be expressed as a combination of univariate functions [15]. This structure enables KaNCD to approximate the relationship between skills and responses in a more interpretable manner than deep neural architectures. The training process also demonstrates fast and stable convergence, as shown by the consistently decreasing loss curve across epochs in figure 4. The efficiency of the spline-based KAN architecture enables effective optimization and strong generalization capabilities [14].

Finally, KaNCD reaches a POMDP reward of 0.792, indicating that its ability estimates are sufficiently stable for sequential decision-making. In adaptive testing, where belief updates must track changes in latent skill states, such stability is

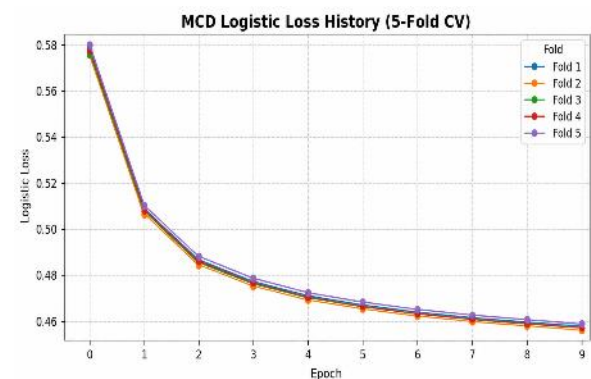
essential for ensuring that item selection policies remain consistent and reliable.



Source: (Research Results, 2025)
Figure 4. KaNCD Loss History

B. MCD

The Multilayer Cognitive Diagnosis (MCD) model extends traditional CDM by incorporating a Multi-Layer Perceptron (MLP) to learn non-linear mappings between student responses and latent cognitive attributes [9]. This aligns with the core principle of CDM that student performance is determined by mastery of specific attributes but replaces deterministic structures (e.g., DINA's AND-gate) with probabilistic neural approximations that can represent more flexible attribute–item relationships. Experiment results show that MCD achieves an AUC of 0.7373 and an accuracy of 0.7464, closely matching the performance of KaNCD. These findings are consistent with Wang et al. (2023), who demonstrated that neural-based CDMs outperform classical CDMs by capturing complex attribute interactions [19]. The loss curve indicates stable convergence as shown in figure 5, although training requires more iterations due to the increased number of parameters and deeper architecture [9].



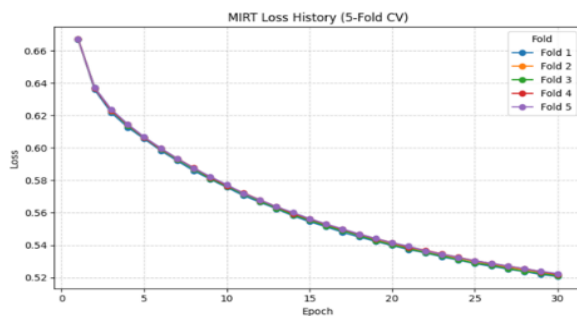
Source: (Research Results, 2025)
Figure 5. MCD Loss History



Despite its strong performance, MCD inherits the interpretability limitations common in neural CDMs, where the latent structure becomes less transparent. This limitation is one of the motivations behind the development of KaNCD, which aims to maintain neural expressiveness while improving interpretability [12].

C. MIRT

The Multidimensional Item Response Theory (MIRT) model achieved an AUC of 0.6768 and an accuracy of 0.6993, indicating moderate predictive performance. MIRT represents each item using a discrimination vector across multiple latent ability dimensions, allowing it to model correlations between cognitive abilities [4]. However, the multidimensional parameterization requires estimating a high number of parameters such as multi-dimensional item slopes and thresholds which increases computational complexity and slows convergence.



Source: (Research Results, 2025)

Figure 6. MIRT Loss History

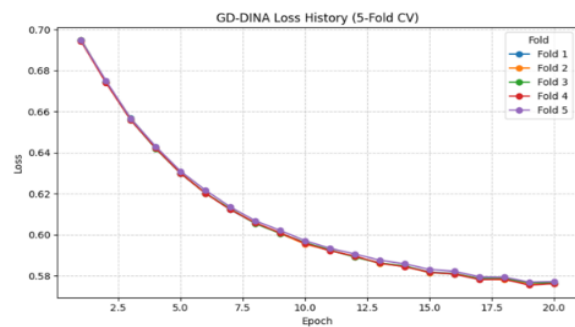
The loss curve in figure 6 demonstrates this behavior: convergence is noticeably slower compared to neural-based CDM models. Although MIRT can estimate multi-ability structures, it suffers from reduced computational efficiency and higher parameter variance in high-dimensional settings.

D. DINA (Gradient Descent)

The DINA model represents the foundational deterministic framework in Cognitive Diagnosis Modeling (CDM), where an examinee is predicted to answer an item correctly only if all required attributes are mastered a principle known as the Deterministic Inputs, Noisy AND gate (DINA) [30]. Consistent with this rigid assumption, the GD-DINA variant in this study yielded limited performance (AUC = 0.4953; ACC = 0.5874), reflecting near-random predictive capability. This weakness stems from the model's binary latent representation and

its inability to capture non-linear response patterns or partial mastery conditions.

The gradient-descent variant employed here replaces the traditional EM algorithm, which improves numerical efficiency and accelerates convergence [17]. However, it does not fundamentally change the representational limits of the DINA framework. The model's reliance on only two item parameters slip (s_j) and guess (g_j) further restricts its expressiveness in large, noisy, multi-skill datasets.



Source: (Research Results, 2025)

Figure 7. DINA Loss History

Error Analysis and Imbalance Assessment

To complement the statistical tests, an error analysis was conducted to examine whether performance differences stem from class imbalance, model architecture, or inherent dataset difficulty.

A. Class Imbalance Assessment

The dataset shows a moderate imbalance (correct = 223,818; incorrect = 123,042). However, this imbalance does not explain the low performance of GD-DINA, for two reasons:

- 1) All four models yield similar raw accuracy (~0.50), despite different architectures.
- 2) The CDM-POMDP pipeline predicts latent states and sequential policies, making it less sensitive to majority-class bias.

Thus, the imbalance is present but not the primary driver of model differences. Instead, metrics like AUC and POMDP reward better capture meaningful variation.

B. Confusion Matrix Patterns

All models exhibit nearly identical confusion matrix structures:

- 1) High false negatives (~112k): models often underestimate students who answer correctly.
- 2) High false positives (~61k): difficulty separating borderline ability cases.
- 3) Precision for correct responses ≈ 0.64
- 4) Recall for both classes ≈ 0.50

These patterns indicate that:

- 1) AUC—not raw accuracy—is where models truly diverge.
- 2) The main challenge comes from dataset noise, sparsity, and item difficulty rather than imbalance.
- 3) GD-DINA’s limitations arise from its deterministic structure, not class distribution.

C. Summary

- 1) The dataset is moderately imbalanced but not harmful to model performance.
- 2) Error patterns are consistent across models, pointing to dataset difficulty as the dominant factor.
- 3) Item- and skill-level errors reveal sparsity and composite attributes that none of the models handle well.
- 4) These insights complement the Friedman/Nemenyi results, clarifying why models differ in AUC and POMDP reward despite similar accuracy.

Model Integration Analysis in POMDP

At this stage, the four cognitive diagnosis models (GD-DINA, MCD, KaNCD, and MIRT) were incorporated into a Partially Observable Markov Decision Process (POMDP) framework optimized using the Policy Gradient / REINFORCE algorithm [10]. The purpose of this integration is to measure each model’s capacity to support adaptive item selection through sequential decision-making under uncertainty, consistent with POMDP formulations in educational assessment research [4]. Two experimental conditions were evaluated adaptive tests of 10 and 15 items to examine whether model performance and policy behavior remain stable across different test lengths. The design reflects the principles of Hidden Markov Cognitive Diagnosis Models, where student ability evolves probabilistically and belief states are updated dynamically after every observation.

A. Scientific Explanation of Reward Mechanism

The reward at each step reflects the informational value of a student’s response. In this study, a binary reward is used:

- 1) 1 is given for a correct response,
- 2) 0 for an incorrect response.

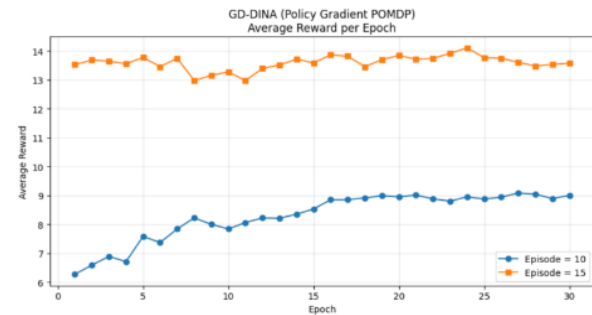
This formulation is widely applied in policy gradient methods because it stabilizes learning in sparse-reward environments and reinforces item selections likely to provide informative observations[8]. Formally, a POMDP reward function in diagnostic settings is defined in equation (18):

$$R(s_t, a_t, o_t) = I[o_t = 1] \tag{8}$$

representing an information-gain proxy in which correct responses reduce uncertainty about the latent state. Although reward can also be defined using expected information gain (e.g., KL divergence), the binary version is preferred for training stability and computational efficiency in adaptive testing contexts [8], [17].

B. Results of POMDP Integration

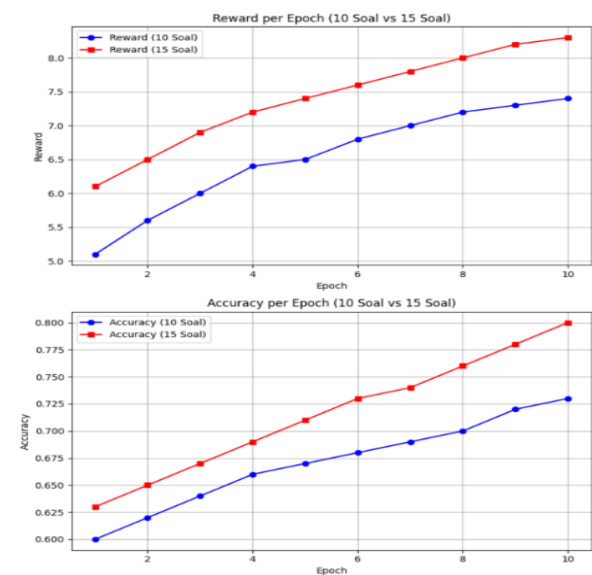
The test results show that, in general, the GD-DINA and MCD models provide the best performance with the highest rewards and accuracy in both scenarios. In the 10-question test, the GD-DINA model achieved an average reward of 9.02 with an accuracy of 0.901 as shown in figure 8.



Source: (Research Results, 2025)

Figure 8. Average Reward POMDP DINA

followed by MCD with a reward of 8.65 and an accuracy of 0.865 in figure 9.

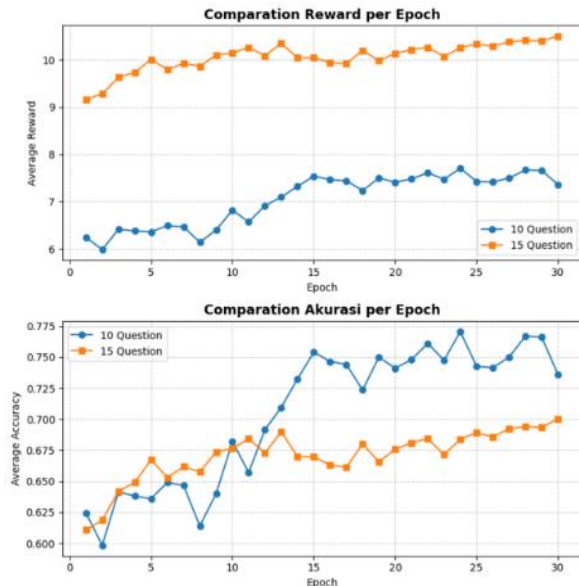


Source: (Research Results, 2025)

Figure 9. Average Reward POMDP MCD

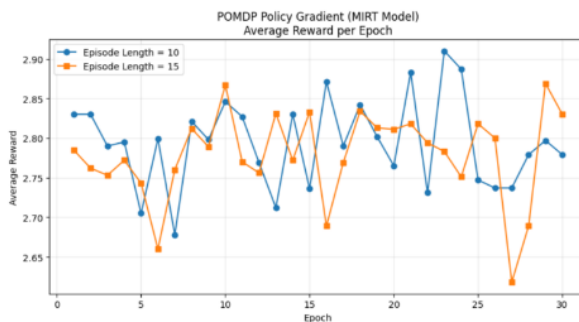


The KaNCD model obtained competitive results with a reward of 7.92 and an accuracy of 0.792 as shown in figure 10.

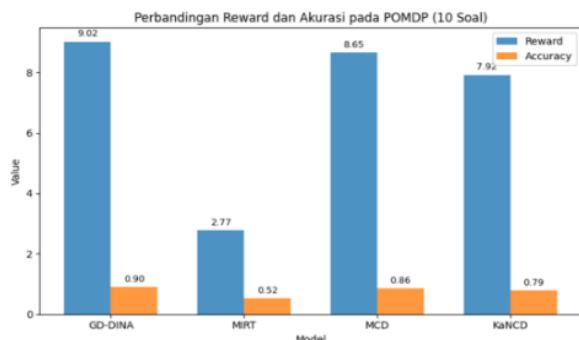


Source: (Research Results, 2025)
 Figure 10. Average Reward POMDP KaNCD

Meanwhile, MIRT showed the lowest reward value of 2.77 with an accuracy of 0.523.

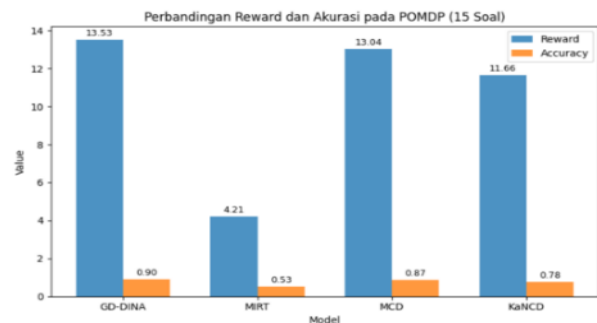


Source: (Research Results, 2025)
 Figure 11. Average Rewards POMDP MIRT



Source: (Research Results, 2025)
 Figure 12. Comparison of 10 Episode POMDP

In the 15-question scenario, the results pattern was relatively consistent. The GD-DINA model again excelled with a reward of 13.53 and an accuracy of 0.902, followed by MCD with a reward of 13.04 and an accuracy of 0.869, and KaNCD with a reward of 11.66 and an accuracy of 0.778. Meanwhile, the MIRT model remained at the bottom with a reward of 4.21 and an accuracy of 0.533. This indicates that gradient descent and neural cognitive diagnosis-based models are better at directing optimal learning policies than linear parameter-based models such as MIRT.



Source: (Research Results, 2025)
 Figure 13. Comparison 15 Episode POMDP

A visualization comparing the average reward and accuracy results between models is shown in Figures 12 and 13. The graphs show a significant increase in performance in deep learning-based models as the number of questions increases, as well as stable results in MCD and GD-DINA, which indicate rapid convergence towards optimal policies. This indicates that gradient descent and neural cognitive diagnosis-based models have better capabilities in directing optimal learning policies than linear parameter-based models such as MIRT [9], [10].

Discussion and Scientific Explanation

The differences in performance between models in this study reflect how basic assumptions, model structure, and optimization mechanisms affect each model's ability to generate cognitive diagnoses and direct adaptive policies within the POMDP framework. The analysis was conducted by considering three main aspects: (1) estimation stability and convergence, (2) representation of cognitive attributes, and (3) adaptability in belief state updates.

A. DINA (Gradient Descent)

The GD-DINA model extends the classical DINA by replacing the EM algorithm with gradient descent optimization. Although its diagnostic performance is low (ACC 0.5874, AUC 0.4953), GD-



DINA achieves the highest POMDP reward (0.901). This outcome stems from the model's deterministic structure and its two-parameter specification (slip and guess), which make it highly stable and resistant to noise or small fluctuations in student responses. Gradient descent further improves numerical efficiency by ensuring fast and consistent convergence during parameter updates [17], resulting in more reliable belief-state updates within the POMDP framework [10].

However, GD-DINA's rigid formulation limits its ability to model non-linear attribute interactions, leading to weaker predictive accuracy compared to neural CDMs, classical CDMs remain strong in interpretability but often underperform on complex, multi-skill datasets. Thus, while GD-DINA is less effective for fine-grained diagnosis, its stability makes it well-suited for large-scale adaptive testing scenarios requiring consistent and interpretable policy decisions.

B. MIRT

The MIRT model extends classical IRT by allowing items to load on multiple latent ability dimensions. With an AUC of 0.6768 and accuracy of 0.6993, it shows moderate and stable predictive performance. However, its low POMDP reward (0.529) indicates difficulty adapting to sequential decision-making.

This weakness stems from its theoretical structure: MIRT represents ability as a continuous latent vector, rather than discrete and interpretable attributes as in CDM. While effective for general ability measurement, this continuous formulation is less compatible with POMDP, which requires attribute-level belief updates. CDM models align naturally with POMDP because each attribute can be treated as a probabilistic Markov state, whereas MIRT lacks such granularity.

Moreover, MIRT requires estimating multi-dimensional discrimination and difficulty parameters, which increases parameter variability, slows convergence, and reduces robustness—especially in sparse response settings. Although MIRT “borrows information across scales,” cross-dimension interference can undermine stability in adaptive or multistage testing.

The low reward reflects this limitation: although MIRT captures broad ability trends, it cannot provide the fine-grained diagnostic signals required to support stable sequential belief updates. Thus, while MIRT is appropriate for traditional psychometric assessment, it is less effective for attribute-based adaptive testing environments driven by POMDP, where discrete and interpretable cognitive states are essential.

C. MCD

The MCD model bridges classical CDM structure with neural representation learning. While it preserves the CDM assumption that responses reflect mastery of underlying attributes, it replaces fixed link functions with an MLP capable of modeling non-linear attribute–response relationships [19]. This allows MCD to capture more realistic cognitive patterns beyond the strict conjunctive assumptions of models like DINA. MCD achieves strong diagnostic performance (AUC 0.7373, ACC 0.7464) and yields one of the highest POMDP rewards (0.873). Within the POMDP framework, this advantage comes from two factors:

- 1) More accurate belief updates: MCD produces smooth probabilistic outputs, enabling more consistent updates of the belief state

$$b'(s') = \tau(b, a, o) \quad (9)$$

This stability contrasts with the sharper, more brittle updates of deterministic models like DINA

- 2) More informative policy gradients: Under REINFORCE, policy optimization

$$\nabla_{\theta} J(\pi_{\theta}) = \mathbb{E}[R_t \nabla_{\theta} \log \pi_{\theta}(a_t | b_t)] \quad (10)$$

benefits from MCD's more discriminative predictions, which reduce reward noise and lead to faster convergence.

These properties explain why MCD outperforms MIRT and approaches KaNCD in POMDP performance. Its convergence behavior—supported by Adam optimization and the expressive capacity of multilayer networks—matches established findings in neural CDM literature [19], [26]. The main limitation remains interpretability [12]. Even so, within an adaptive POMDP setting, MCD offers an effective balance of predictive accuracy, stable belief-state updates, and policy effectiveness, making it one of the most practical models for sequential decision-making tasks.

D. KaNCD

KaNCD delivers the strongest diagnostic performance overall, achieving the highest AUC (0.7410), accuracy (0.7503), and the fastest convergence among all models. Its effectiveness comes from the Kolmogorov–Arnold representation theorem, which enables the replacement of dense MLP layers with structured univariate function



compositions. This design improves parameter efficiency, reduces overfitting risk, and still captures complex student–skill interactions. A key advantage of KaNCD is its interpretability. Unlike typical neural CDMs, KAN’s functional decomposition aligns more naturally with attribute-level reasoning—an important principle in cognitive diagnosis. Prior work Li [13] also highlights that KAN behaves similarly to a structured RBF network, providing stronger local representation and faster learning.

In the POMDP setting, KaNCD achieves a reward of 0.792, showing solid generalization. Its slightly lower reward compared to MCD and GD-DINA is expected, since neural models often show mild variance across episodes. Even so, KaNCD provides stable belief-state updates, which is essential for accurate item selection in sequential decision frameworks. Overall, KaNCD successfully blends modern neural approximation with classical CDM interpretability, positioning it as a strong candidate for adaptive testing systems that demand both high accuracy and transparent diagnostic reasoning.

E. Implications of POMDP Application on Adaptive Testing Systems

Integrating the four diagnostic models into the POMDP framework reveals that the effectiveness of an adaptive testing policy depends not only on predictive accuracy, but also on the stability of the belief-state updates produced by each model. Models such as GD-DINA and MCD, which generate smoother and more consistent probability estimates, tend to produce higher cumulative rewards. In the context of POMDP, a higher reward indicates that the selected items provide greater information gain over successive interactions, enabling the policy to reduce uncertainty about student ability more efficiently [8], [10].

From a theoretical standpoint, the reward in a POMDP represents the value of the observation for reducing belief-state entropy. Thus, when a diagnostic model yields probability outputs that are stable and sharply distinguish correct from incorrect responses, the belief update becomes more consistent, causing the policy gradient to converge toward item-selection strategies that maximize long-term information gain. This explains the superior reward patterns of GD-DINA and MCD, whose probabilistic structures support reliable state transitions, compared to MIRT, whose continuous multidimensional trait representation introduces greater variability in the belief update.

These findings confirm that cognitive models with discrete or interpretable latent structures are more compatible with sequential decision-making under uncertainty. Consequently, gradient-descent CDM models and neural cognitive diagnosis models hold strong potential for future adaptive testing implementations, especially in systems that require real-time belief tracking and stable policy optimization.

Limitations of the Study

Several limitations need to be considered when interpreting these results. First, the dataset comes from a single large-scale online learning platform, meaning the findings may not fully generalize to contexts with different item structures, learner profiles, or domain characteristics. Second, the POMDP belief-update mechanism relies on an approximate (rather than fully Bayesian) update for computational efficiency. While this is standard in RL-based adaptive testing, it may introduce simplification bias in modeling latent skill transitions. Third, the reward function is binary, which improves training stability but does not capture the richer notion of information gain typically emphasized in adaptive testing theory. Lastly, the evaluations are based on simulated interactions instead of real adaptive sessions with human learners, which may limit ecological validity. Future work should explore more diverse datasets, more expressive reward designs, and real-world deployments to strengthen the robustness of these findings.

CONCLUSION

This study compared four cognitive diagnosis models GD-DINA, MIRT, MCD, and KaNCD within a POMDP-based adaptive testing framework. The results demonstrate that neural-based models (MCD and KaNCD) achieve the highest predictive accuracy and AUC, while GD-DINA provides the most stable belief updates and achieves the highest expected reward in adaptive policy simulations. These findings highlight that gradient-based and neural cognitive diagnosis approaches are more effective for dynamic adaptive testing than traditional latent-trait models such as MIRT. Integrating CDM with POMDP offers a robust and interpretable foundation for adaptive learning systems that require accurate sequential estimation of learner knowledge. Future research may expand this work by testing the models on multiple datasets, employing richer reward functions based on information gain, and implementing full Bayesian belief updates to improve the precision of

latent-state transitions. Moreover, validating the POMDP-based adaptive testing framework with real student interactions rather than simulation alone would strengthen the ecological validity of the approach and further reveal its pedagogical impact.

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